

205 – OPERATIONS RESEARCH THROUGH EXCEL SOLVER

UNIT – I

Introduction to OR: Definition of OR, Managerial Applications of OR, Linear Programming Problem – Model Formulation, Graphical Method, and Simplex Method – Linear Programming Using the Excel Solver.

UNIT – II

Transportation Problem: Balanced Transportation Problem and Degeneracy – Transshipment Model – Assignment Model: Hungarian Method, Salesmanship Model, Maximisation and Minimisation using Transportation Models.

UNIT – III

Dynamic Programming: Features, Structure and Applications; Dynamic Programming Algorithm; Solving LPP using Dynamic Programming.

UNIT –IV

Goal Programming: Meaning and Concept of Goal Programming – Application Areas, Model Formulation, Modified Simplex Method– Advantages – Procedure (Theory).

UNIT – V

Queuing Theory Model: Components, Basic Structure Assumptions – Waiting Lines Decision Problem – Simulation: Simulation Methodology, use of Random Numbers – Applications of Simulations.

Introduction

Making a decision is becoming more and more complex. One has to take certain decisions for himself and for others. Managers are evaluated primarily on the results of their decisions and successful managers use Quantitative approaches to decision making.

These approaches are also known as

- * Operations research (OR) - scientific process
- * Management Science (MS)
- * Decision Science (DS)
- * Quantitative techniques for Business Decisions (QTBDD).

Operation research is an application of scientific approach to solve management problems in order to help managers to make better decisions.

An effective decision depends on many factors, which may be an economic, social and political.

eg: Starting of a new factory of a place would depends on economic factors

1. Construction cost
2. Labour cost
3. Availability of raw materials
4. Transportation cost
5. Taxes.

6. Energy cost

7. pollution control cost etc.

Qualitative and Quantitative Analysis :- (1.1.1)

While solving a real life problem the decision maker must examine a problem from Both Qualitative and Quantitative prospective, so that data, so obtained should be analysed in both Prospectives. to suggest solution of a problem

Qualitative Analysis :- It is primarily based on the Judgement and experiences of the manager and it includes the managers feelings regarding the problem.

Qualitative Analysis is emphasized in the situations where the managers has experiences with similar problems or if the problem is relatively simple

Quantitative Analysis :- It is recommended, if the problem is Quite complex and the manager is not having enough. Experience with similar problems.

Quantitative Analysis are primarily mathematical and Statistical Techniques are used in managerial decision making.

Quantitative approach to problem Solving requires collections of large amount of data and development of mathematical models.

Quantitative Techniques are used to support management to make decision under complex and uncertain Situation In order to achieve the object and goals of the organization.

Consider a problem of an investor seeking advice for investment in 3 alternatives (stock market, Real estate and Bank deposit). To suggest an acceptable solution we need to consider certain Quantitative factors to be examined in the light of the problem.

(1.2) Origin and Growth / Evaluation / History of OR:-

— However a particular model and Technique of O.R was used as early as in World War-1.

→ In the year 1914 - 1915 Thomas Edison used a tactical game board to minimize shipping losses from enemy submarines instead of risking ships in actual war conditions. (Optimum utilization of resources, Decision making)

→ About the same time, A.K. Erlang developed experiments to study the fluctuations in demand for telephone facilities using Automatic dialing equipment, later on such experiments were used for the development of waiting line theory.

Stage 1 During world war II :- It is generally agreed that 'OR' came into existence during world war-II when there was a critical need manage scarce resources in military operations. Since the war involved strategic and tactical problems where greatly complicated and provided adequate solutions.

Therefore groups of individuals who collectively were considered has specialist in mathematics, Economics, Statistical and probability theory, Engineering, Behavioural and physical science where found a special units within the armed force to deal with strategic and tactical problems of various military operations.

Such groups were first formed by the British air force and later the America armed force. Similar groups.

1. At very first Britain developed Radar operations Research unit and was assigned to analyse the problem of Radar at gun sites under the guidance of prof. pms Blackett which also sites under the guidance of prof. p. called as Air Battle of Britain.

2. Later United States Canada and France started the use of OR Techniques which with such team of Scientist.

3. At the end of world war-II, the scientist of this team moved to different sectors such as Transportation, Health, Education.

Stage 2 After world war-II:-

After the war ended, OR approaches applied to civilian problems, Business, Research and Development.

There are various important factors behind the rapid development in the use of operations research approach

→ After world war-II, the economic and industrial Boom resulted in continuous automachine, mechanization decentralisation of operations and division of management function and resulted in complex managerial problems.

There Applications of OR to managerial Decision making became popular.

→ In the year 1947, George B. Dantzig, developed the concept of linear programming to allocate scarce resources by a method known as simplex method.

→ At the end of 1950 many other techniques of OR developed such as S.O.C, Dynamic programming, Queuing theory and inventory theory.

→ A year later project scheduling techniques C.P.M and PERT were developed as efficient tools for scheduling and monitoring lengthy, complex and expensive projects of that time.

which later changed its name to the O.R. Society of U.K. Its Journal O.R. Quarterly first appeared in 1950.

→ In 1952 the O.R. Society of America (O.R.S.A) founded and its Journal operations research was first published in 1953.

→ In 1953 the Institute of Mgt Science (IMS) was founded as an International Society to identify, extent and unify scientific knowledge pertaining to management. Its Journal management science first appeared in 1954.

Stage 3. In India :-

In India O.R. Originated during the last phase of world war II. Initially an O.R. Centre was formed at the Air. Head, Quarterly to tackle problems relating to Radar.

→ In India O.R. started in 1949. with the opening of a unit at the regional research, laboratory, Hyderabad. for using O.R. in planning and organising research.

→ In 1953 an O.R. Unit was found at the Indian Statistical Institute, Calcutta with a view to apply O.R. Techniques to National planning under the guidelines of "Prof. Mahalanobis".

→ O.R. got its formal recognition in the country with the formation of O.R. Society of India (O.R.S.I) in 1957. The first president of the society "P.C. Mahalanobis".

→ The society became the member of International Federation of O.R. Societies in 1959 and organised its first conference at all India level during the same year.

→ The publication of journal "O.P.S.R.A.R.C.H." (OPSEARCH) by O.R.S.I in 1954, further added to the growth of O.R activities in India. An International Seminar "second O.R around the world meeting" Sponsored by O.R.S.I was held at New Delhi during August 7-10th in 1967.

→ Today however services Organisation such as banks, Hospitals, Libraries, Airlines, Railways etc., recognised the usefulness of O.R in improving efficiency because of O.R's multi-disciplinary character and application in varied fields. It has a bright future.

Nature / characteristics of O.R. :-

Operation Research can be considered as being the application of scientific method by interdisciplinary teams to problems involving the control of organised (man, machine) systems so as to provide solutions which are best for the purpose of the organization of a whole.

1) Interdisciplinary Team Approach :- The O.R has the characteristics that it is done by a team of scientist draw from different disciplines such as mathematics, statistics, Economics, Engineering, physics etc. It is essentially an interdisciplinary team approach.

2) System Approach :- O.R emphasis on the overall approach to the system. This characteristic of the O.R is often referred as a system oriented the Orientation. It is based on the observation that in

ultimately has same effect on other part.

O.R. cannot give perfect answers (or) solutions to the problems. It merely bad answers to the problems which otherwise have worst answers. Thus O.R. simply helps in improving the quality of solutions but does not result into a perfect solution.

3). Goal Oriented Optimum solution :- O.R. tries to optimise a well defined function subject to give consigns and as such as consign with the Optimization theory.

4). Scientific Method :- O.R. involves scientific and systematic attack of complex problems to arrive at optimum solutions. In other words O.R. uses techniques of scientific research.

5). Use of models :- O.R. use models build by Quantitative measurements of the variable concerning a given problem and also derives a solution from the model using one or more of the diversified solution techniques.

Definitions of O.R :-

1. O.R is a scientific approach to problem solving for executive management.

- H.M. WAGNER.

2. OR may be described as a scientific approach to decision making that involves the operations of Organisational system.

- F.S. Hillier and G.J. Lieberman.

3. O.R is a systematic application of Quantitative methods, Techniques and Tools to the analysis of problems involving the operations of systems.

4. O.R is essentially a collection of mathematical Techniques and Tools which in conjunction with a system approach, are applied to solve practical decision problems of an economic or engineering Nature. — George.

5. OR is the application of scientific methods, Techniques and Tools to problems involving the Operations of systems so as to provide those in control of operations with optimum solution to the problems.

— CHURCHMAN

6. OR is the Art of finding bad answers to problems which otherwise worse answers.

SCOPE / APPLICATIONS OF OR

OR is today recognised as an applied Science concerned with large number of diverse human activities to the precise on operation uses some valuable resources like man, money, machine, Time, effort etc.

— An OR worker is required.

1. To minimize the output value for a specific output.
2. To maximize the output value for a specific input.
3. To maximize some functions of these values.

Ex:- To profit or return on investment some of the areas of management where techniques of OR are applied is listed below.

In Agriculture:- with the explosion of population and consequent shortage of food, Quantitative Techniques are very useful in Sorting out problems.

* The problem of optimum distribution of water from various resources, fertilizers, new techniques and capabilities.

* The problem of optimum allocation of land in various crops in accordance with the climate conditions.

In finance :- In this modern times of economic crisis, it has become very necessary for every government to have careful planning using the Quantitative Techniques applied to:

* maximize the per capita income with minimum resources.

* find out the profit plan for the company.

* Determine the best replacement policies.

In Industry :- If the industry manager decides his policies only on the basis of his past experiences, he may not lead the organization in a positive manner and organization will suffer heavy losses. This losses can be compensated by using Quantitative Techniques in Business management with formulates total production schedule, optimum utilization of resources available.

In marketing :- with the help of OR a marketing manager can decide where to distribute the product for sale so that total carriage inwards and outwards cost will be minimised.

* Minimum per unit sale price

* Size of the stock to meet the future demand.

* Selection of advertising media with respect to time, cost, efforts, results etc.

* purchasing criteria at minimum possible cost.

Personal management

A personal manager can use OR

- * To appointment the most suitable persons an appropriate salary.
- * To determine the best age of retirement.
- * To find out the number of persons to be appointed on fulltime basis depending upon the nature of the work load.

In production management :- A production manager can use OR.

1. To find the number and size of the items to be produced.
2. In scheduling and sequencing the production run by proper allocation of machines.
3. In calculating the optimum product mix.
4. To select, locate and design production plants.

In Insurance Sector :-

- * What should be the premium rates for various modes of policies.
- * How best the profits could be distributed in the case of profit generation.

Limitations of OR :-

The limitations of OR are mostly related with the problems concerned with model building and the time and cost factors involved in its applications rather than its practical utility. Some of these limitations are.

1. OR provides the solutions only when the problem consists of only quantifiable elements. But in real situations all relevant variables are

influencing factors cannot be quantified and some may be qualitative in nature. factors which cannot be quantified cannot be used in the mathematical models used in OR.

2. OR tries to find out the optimal solution, considering all the factors individual in a problem. In present days business problems are effected by a number of factors. and expressing them with one another requires enormous calculations. such calculations cannot be handled manually and require electronic computers involving heavy cost. thus makes applications of OR suitable to only very large organisations.

3. Application of Mathematics model of OR requires persons having knowledge of mathematics or statistics who might not be aware of the problems related to business. managers who can understand business problems fail to understand the complexity of OR.

4. Application of OR becomes a costly affair when the basic data is subjected to frequent changes also a fairly good solution which can be implement at present may be better than an optimal solution which may be possible to implement at a late date.

5. Some times implementation of decisions may becomes a delicate tasks because of resistance offered by people who are affected by the Decision. Hence application of OR models should take into account complexities of human relations and Behaviour.

Phases of OR :-

There are three phases of the scientific model on which approach is based.

Judgement phase

This phase include

- (i) Identification of the real life problem.
- (ii) Selection of an appropriate objective and Selection of various variables related to this objective
- (iii) Application of the appropriate scale of measurement that is measures of effectiveness.
- (iv) formulation of an appropriate model of the problem.

2. Research phase :- This phase is the largest and longest among other two phases this phase utilises observations and data collections for a understanding of the problem.

- (i) formulation of hypothesis and model.
- (ii) Observation and experimentation to test the hypothesis.
- (iii) Analyse of the available information and verification of the hypothesis using pre-established measures of desirability.
- (iv) prediction of various results from the hypothesis.
- (v) Generation of the result and consideration of alternative method.

3. Action phase :- This phase consist of

- (i) making recommendations for implementing the decision by an individual who is in the position to implement result.
- (ii) This individual must be aware of the environment in which the problem occurred, objective, assumptions and omissions of the models of the problem.

Introduction :- The term linear programming is a combination of two words linear and programming. The term linear refers to linear relationship among the variables in a model. Thus a given change in one variable is always cause a resulting proportional change in another variable. The word programming refers to modeling and solving problem mathematically that involves the economic allocation of limited resources by choosing a particular course of action or strategy among various alternative to achieve the desire objectives.

Linear programming is a mathematical modeling technique useful for the allocation of source are limited resources such as men, machine, material, time, warehouses, capital etc., to several competing activities such as products, services jobs, new projects on the basis of a given criteria of optimality.

Structure and model of LPP :-

The general structure of LPP consists of three basic components. they are.

1. Decision Variables :- The LPP can be applied to the given problem contains various alternatives (courses of actions) for arriving the optimal value of objective function. These alternatives are evaluated by nature of objective function and availability of resources for this we study. certain activities those are called decision variables. usually denoted $x_1, x_2, x_3, \dots, x_n$
Ex:- The product mix contains three products a, b, c and the number of units for product a is x_1 , for product

b is x_2 and c_1, x_3 are the decision variables.

The values of these variables may or may not be under the decision makers, then such variables are said to be controllable; otherwise uncontrollable.

But in an LPP all decision variables are controllable, continuous and non-negative, i.e., $x_j \geq 0$, $j = 1, 2, \dots, n$.

2. Objective function:- The objective function of each LPP is expressed in terms of decision variables to optimize (maximize/minimize).

The criteria of optimality such as profit, cost, revenue, distance etc. it is represented as optimize.

$$\text{Optimize [Maximum or minimum]} z = \sum_{j=1}^n c_j x_j$$

$$= c_1 x_1 + c_2 x_2 + c_3 x_3 + \dots + c_n x_n$$

where z = The optimality, which is a function of Decision variables x_j ; $j = 1, 2, 3, \dots, n$.

c_j = parameters that represent the contribution of a unit of the respective variables $x_1, x_2, x_3, \dots, x_n$.

Constraints :-

In every Organization certain limitations are constraints are available on the use of resources that is men, machine, material, Time etc. that limit the degree to which an Objective can be achieved.

Such constraints must be expressed as linear equalities or inequalities in terms of Decision variables.

Ex:- Consider the labour constraint so a limited labour for three products is so for three products the decision variables are considered as x_1, x_2, x_3 and labour constraint is $x_1 + x_2 + x_3 \leq 50$.

Mathematical Model of LPP :-

The General mathematical model of LPP contains three components as follows.

Objective function :-

Optimize [Maximize (or) minimize] $Z = \sum_{j=1}^n C_j x_j$

$$= C_1 x_1 + C_2 x_2 + C_3 x_3 + \dots + C_n x_n$$

Subject to constraints :-

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n [\leq, =, \geq] b_1$$

$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n [\leq, =, \geq] b_2$$

$$a_{i1} x_1 + a_{i2} x_2 + \dots + a_{in} x_n [\leq, =, \geq] b_i$$

$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n [\leq, =, \geq] b_m$$

Non-Negativity restrictions :-

$$x_j \geq 0 ; j = 1, 2, 3, \dots, n$$

Formulation of LPP:- The following steps to be followed while formulating in LP problem.

Step ① - Identifying Decision variables.

→ firstly identify number of products 'n'.
Let the decision variables $(x_j \geq 0; j=1, 2, \dots, n)$ for 'n' products.

Ex:- A, B are two products in the given below.
Let x_1 and x_2 are number of units of products A and B.

Step ② - Identify the problem data.

→ Identify the contribution (co-efficient) of decision variables for both objective function c_j and constraints a_{ij} .

Step ③ - formulate the objective function.

→ Identify whether the objective function is to be maximized or minimized, then formulate the objective function into a linear mathematical expression in terms of Decision variables multiplied by their profit or cost contribution (co-efficient of Decision Variables c_j).

$$\text{Maximize or minimize } Z = \sum_{j=1}^n c_j x_j = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

Step ④ - formulate the constraints.

→ Express the constraints verbally in terms of requirements, availability and equals to each resource.

Step ⑤ - formulate LPP.

→ finally formulate LPP as follows.

objective function:- maximize (or) minimize $Z = \sum_{j=1}^n c_j x_j$

Subject to constraints:- $\sum_{j=1}^n a_{ij} x_j \leq \text{or } = \text{or } \geq b_i; i=1, 2, \dots, m$

Restrictions : $x_j \geq 0$; $j = 1, 2, \dots, n$.

Complete model of LPP : $5x_1 + 10x_2 = 200$

Max $Z = 250x_1 + 400x_2$ - objective function

$$S.T. \leq 2x_1 + 5x_2 \leq 200$$

$$4x_1 + 2x_2 \leq 240$$

$$x_1, x_2 \geq 0$$
 - non-negativity restrictions.

2-16/22

Problems :- on LPP

1. A company manufactures two types of products P_1 & P_2 . Each product uses lathe and milling machines. The processing time per unit of P_1 on the lathe is 5 hrs and on the milling machine is 4 hrs. The processing time per unit of P_2 on the lathe is 10 hrs and on the milling machine is 4 hrs. The maximum number of hours available per week on the lathe and milling machines are 60 hrs and 40 hrs respectively. Also the profit per unit of selling P_1 and P_2 are ₹.6 and ₹.8. Respectively, formulate a linear programming model to determine the production volume of each of the product such that the total profit is maximized.

Sol:-

Machine type	Processing Products type		avail-able hrs.
	P_1	P_2	
lathe	5	10	60
milling	4	4	40
Profit	26	28	

Objective functions:

$$\text{Max } Z = 6x_1 + 8x_2$$

Subject to constraints:

$$\text{Lathe} \Rightarrow 5x_1 + 10x_2 \leq 60$$

$$\text{Milling} \Rightarrow 4x_1 + 4x_2 \leq 40$$

Non-negativity Restrictions

$$x_1, x_2 \geq 0$$

- ③ A manufacturer produces two types of models m_1 & m_2 . Each model of type m_1 requires 4 hrs of polishing and 2 hrs of grinding where each of the model m_2 requires 2 hrs of polishing and 5 hrs of grinding. The manufacturer have two polishers and 3 grinders. Each polisher works 40 hrs a week and Each grinder works 60 hrs a week. Profit on m_1 is ₹3 and m_2 is ₹4. Whatever is produced in a week is sold in the market. How should the manufacturer allocate this production capacity to the two types of model so that he make maximum profit in a week

Machine type	M_1	M_2	Available hrs.
Polishing	4	2	$40 \times 2 = 80$
grinding	2	5	$60 \times 3 = 180$
Profit	₹ 3	₹ 4	

Objective function

$$\max Z = 3x_1 + 4x_2$$

Subject to Constraints

$$\text{Polishing} \Rightarrow 4x_1 + 2x_2 \leq 80$$

$$\text{Grinding} \Rightarrow 2x_1 + 5x_2 \leq 180$$

Non-negativity Restrictions

$$x_1, x_2 \geq 0$$

- ③ A person requires 10, 12, 12 units chemicals a, b, c respectively for his garden. A liquid product contains 5, 2, 1 units of a, b & c respectively. A dry product contains 1, 2, 4 units of a, b & c with the liquid product sells for ₹ 3 & dry product sells ₹ 2. how much of each should be purchased in order to reduce the cost and meet the requirements.

Solution of linear programming problems:

LPP can be solved basically by three ways

- 1) Graphical method
- 2) Simplex method
- 3) Transportation method.

Graphical Method

Solving LPP using graphical method Procedure

* The industrial problems involving two or three variables can be easily and effectively solved by drawing the graph

for various constraints and objective function.

It is even simpler for two variable cases as three dimensional geometric is rather difficult to draw on graph paper.

Steps for graphical solution of LPP.

Step:1 The constraints of the problem are taken as equalities and plotted on the graph paper. The region satisfying each

Constraint is determined region common to all the constraints. relations is located. this region is known as feasible region for the problem as all points lying in this region will simultaneously satisfy the given constraints.

→ The solution set of the system of constraints is found to be represent by some convex polygon termed as feasible region.

Step:2 The objective function is also plotted on the same graph paper by taking some arbitrary value of 'Z'.

Step:3 Trial and Error method is used to locate the optimal solution inside the feasible region determined in step 1, so that the objective function is optimized. It is experienced that the optimal solution is located near the extreme points of the feasible region does the optimum feasible point should be such only at the corners of the convex polygon. The graph for all the constraints and the objective function will be straight lines in 2PP.

As the optimal values occurs at the corner points of the feasible region it is enough to calculate the value of objective function of the corner points of feasible region and the select one which gives optimal solution in the case of maximization problem.

Optimal point correspondent to the corner point at which the objective function has a maximum value and, in the case of minimization, the corner point which gives the objective function the minimum value as the optimal solution.

Introduction to maximization.

* In the case of maximization problem optimal point corresponds to the corner point at which the objective function has a maximum value.

* If the inequality constraint corresponding to that line is $\leq$ then the region below line lying in first quadrant is shaded.

* In this case the line of objective function will stop farthest from origin and passing through atleast one corner of feasible region.

28/6/22 Problems on maximization with two constraints

1) solve the LPP by using graphical method

$$\max Z = 12x_1 + 16x_2$$

Sub to constraints

$$10x_1 + 20x_2 \leq 120$$

$$8x_1 + 8x_2 \leq 80$$

$$x_1, x_2 \geq 0$$

Given Equations

$$10x_1 + 20x_2 = 120$$

$$8x_1 + 8x_2 = 80$$

from Eq ①

$$10x_1 + 20x_2 = 120$$

$$\text{when } x_1 = 0$$

$$10(0) + 20x_2 = 120$$

$$20x_2 = 120$$

$$x_2 = \frac{120}{20}$$

$$x_2 = 6$$

x_1	0	12
x_2	6	0

$$\text{when } x_2 = 0$$

$$10x_1 + 20(0) = 120$$

$$10x_1 = 120$$

$$x_1 = \frac{120}{10}$$

$$x_1 = 12$$

from Eq ②

$$8x_1 + 8x_2 = 80$$

$$\text{when } x_1 = 0$$

$$8(0) + 8x_2 = 80$$

$$8x_2 = 80$$

$$x_2 = \frac{80}{8}$$

$$x_2 = 10$$

x_1	0	10
x_2	10	0

$$\text{when } x_2 = 0$$

$$8x_1 + 8(0) = 80$$

$$8x_1 = 80$$

$$x_1 = \frac{80}{8}$$

$$x_1 = 10$$

from the graph,

ABCD is a feasible region

A(0,0), B(10,0), C(8,2), D(0,6).

$$\text{Max } Z = 12x_1 + 16x_2$$

$$A(0,0)$$

$$Z(A) = 12(0) + 16(0) = 0$$

$$Z(B) = 12(10) + 16(0) = 120$$

$$Z(C) = 12(8) + 16(2) = 128$$

$$Z(D) = 12(0) + 16(6) = 96$$

for x_1 and x_2 maximum for the constraint C, $10x_1 + 20x_2 = 120$

$$Z(\text{optimising}) = 128$$

Corner Point

Constraint C, $10x_1 + 20x_2 = 120$

2) Maximisation problem with two constraints
formulate & solve the LPP by using
graphical method.

A & B are two products to be manufactured
Unit profit on A is 40 & B is 35. Maximum
material available is 60kgs and labour 96hrs.
Each unit of A needs 2kgs of material &
3 man hrs, whereas each unit of B needs
4kgs of material & 3 man hrs. find optimum
level of A & B to be manufactured

Sol

formulation of LPP.

	A	B	Max avail hm.
Material	2	4	60
labour	3	3	96
Profit	40	35	

Objective function

$$\max Z = 40x_1 + 35x_2$$

Subject to constraints

$$2x_1 + 4x_2 \leq 60$$

$$3x_1 + 3x_2 \leq 96$$

Non negativity constraints

$$x_1, x_2 \geq 0$$

Solution of LPP.

Step: 1 solve the inequalities into Equations then obtained Equations are

$$2x_1 + 4x_2 = 60 \quad - \textcircled{1}$$

$$3x_1 + 3x_2 = 96 \quad - \textcircled{2}$$

from Eq $\textcircled{1}$

$$2x_1 + 4x_2 = 60$$

when $x_1 = 0$

$$2(0) + 4x_2 = 60$$

$$4x_2 = 60$$

$$x_2 = \frac{60}{4}$$

$$x_2 = 15$$

x_1	0	30
x_2	15	0

When $x_2 = 0$

$$2x_1 + 4(0) = 60$$

$$2x_1 = 60$$

$$x_1 = \frac{60}{2}$$

$$x_1 = 30$$

from Eq $\textcircled{2}$

$$3x_1 + 3x_2 = 96$$

when $x_1 = 0$

$$3(0) + 3x_2 = 96$$

$$3x_2 = 96$$

$$x_2 = 96/3$$

$$x_2 = 32$$

x_1	0	32
x_2	32	0

when $x_2 = 0$

$$3x_1 + 3(0) = 96$$

$$3x_1 = 96$$

$$x_1 = 96/3$$

$$x_1 = 32$$

from the graph

ABC is a feasible region

$$A(0,0) \quad B(30,0) \quad C(0,15)$$

$$\text{Max } Z = 40x_1 + 35x_2$$

$$A(0,0)$$

$$Z(A) = 4(0) + 35(0) = 0$$

$$Z(B) = 40(30) + 35(0) = 1200$$

$$Z(C) = 40(0) + 35(15) = 525$$

for Z value is maximum

for the corner point

$$B, \text{ so } x_1 = 30, x_2 = 0$$

$$Z(\text{optimizing}) = 1200$$

3) Solve the LPP by using graphical method.
Maximisation problem with three constraints

$$\text{Max } Z = 5x_1 + 7x_2$$

Sub to Cont

$$3x_1 + 8x_2 \leq 24$$

$$x_1 + x_2 \leq 4$$

$$10x_1 + 7x_2 \leq 35$$

$$\text{NRP: } x_1, x_2 \geq 0$$

So)

Convert the inequalities into Equalities
then obtained equations are.

$$3x_1 + 8x_2 = 24 \quad - (1)$$

$$x_1 + x_2 = 4 \quad - (2)$$

$$10x_1 + 7x_2 = 35 \quad - (3)$$

from Eq (1)

$$3x_1 + 8x_2 = 24$$

when $x_1 = 0$

$$3(0) + 8x_2 = 24$$

$$8x_2 = 24$$

$$x_2 = \frac{24}{8}$$

$$x_2 = 3$$

x_1	0	8
x_2	3	0

when $x_2 = 0$

$$3x_1 + 8(0) = 24$$

$$3x_1 = 24$$

$$x_1 = \frac{24}{3}$$

$$x_1 = 8$$

x_1	0	4
x_2	4	0

from Eq (2)

$$x_1 + x_2 = 4$$

when $x_1 = 0$

$$0 + x_2 = 4$$

from eq 3

$$10x_1 + 7x_2 = 35$$

when $x_1 = 0$

$$10(0) + 7x_2 = 35$$

$$7x_2 = 35$$

$$x_2 = \frac{35}{7}$$

$$x_2 = 5$$

x_1	0	3.5
x_2	5	0

when $x_2 = 0$

$$10x_1 + 7(0) = 35$$

$$10x_1 = 35$$

$$x_1 = \frac{35}{10}$$

$$x_1 = 3.5$$

from the above graph
 ABCDE is a feasible region.
 A(0,0) B(3.5,0) C(2.5,1.7) D(1.6,2.4) E(0,3)

$$\text{max } z = 5x_1 + 7x_2$$

$$z(A) = 5(0) + 7(0) = 0$$

$$z(B) = 5(3.5) + 7(0) = 17.5$$

$$z(C) = 5(2.5) + 7(1.7) = 24.4$$

$$z(D) = 5(1.6) + 7(2.4) = 8 + 16.8 = 24.8$$

$$z(E) = 5(0) + 7(3) = 21$$

for z value is maximum for the corner

Point D, so $x_1 = 1.6$, $x_2 = 2.4$

$$z(\text{optimising}) = 24.8$$

Introduction to minimisations.

Many linear programming problems involve minimising an object to such as cost instead of maximising a profit function.

For example, a restaurant may wish to develop a work schedule to meet staffing needs while minimising the total number of employees.

* The solution to minimisation problem follows some procedures as that of a maximisation problem. The basic difference is that now smallest value for objective function will be calculated.

⇒ Minimisation problem with two constraints solve the problem LPP by using graphical method

$$\text{min } Z = 4x_1 + 6x_2$$

Sub to Constraints

$$x_1 + x_2 \geq 8$$

$$6x_1 + x_2 \geq 12$$

NRP

=

$$x_1, x_2 \geq 0$$

then the obtained Equations.

$$x_1 + x_2 = 8 \quad \text{--- Eq ①}$$

$$6x_1 + x_2 = 12 \quad \text{--- Eq ②}$$

from 1st Equation.

x_1	0	8
x_2	8	0

$$x_1 + x_2 = 8$$

when $x_1 = 0$

$$0 + x_2 = 8$$

$$x_2 = 8$$

when $x_2 = 0$

$$x_1 + 0 = 8$$

$$x_1 = 8$$

from 2nd Equation.

x_1	0	2
x_2	12	0

when $x_1 = 0$

$$6(0) + x_2 = 12$$

$$x_2 = 12$$

when $x_2 = 0$

$$6x_1 + 0 = 12$$

$$6x_1 = 12$$

$$x_1 = 12/6$$

$$x_1 = 2$$

from the graph.

ABC is a feasible region,
A (8,0), B (0.8, 7.2), C (0,12)

$$\min z = 4x_1 + 6x_2$$

$$z(A) = 4(8) + 6(0) = 32$$

$$z(B) = 4(0.8) + 6(7.2) = 46.4$$

$$z(C) = 4(0) + 6(12) = 72$$

for z value is minimum for corner point 'A' so $x_1 = 8$ & $x_2 = 0$

$$z(\text{optimum}) = 32$$

5) Minimisation problem with three constraints
solve the LPP by using graphical method

$$\min z = 3x_1 + 2x_2$$

Sub to Constraints

$$5x_1 + x_2 \geq 10$$

$$x_1 + x_2 \geq 6$$

$$x_1 + 4x_2 \geq 12$$

N.R.

$$x_1, x_2 \geq 0$$

Sol

Conversion of inequalities into Equalities.
then obtained equations are.

$$5x_1 + x_2 = 10 \quad \text{--- Eq ①}$$

$$x_1 + x_2 = 6 \quad \text{--- Eq ②}$$

$$x_1 + 4x_2 = 12 \quad \text{--- Eq ③}$$

from Eq ①

$$5x_1 + x_2 = 10$$

when

$$x_1 = 0$$

$$5(0) + x_2 = 10$$

$$x_2 = 10$$

$$x_1 \quad 0 \quad 2$$

$$x_2 \quad 10 \quad 0$$

when $x_2 = 0$.

$$5x_1 + 0 = 10$$

$$5x_1 = 10$$

$$x_1 = 2$$

from Eq ②

when $x_1 = 0$.

$$x_1 + x_2 = 6$$

$$x_2 = 6$$

when $x_2 = 0$

$$x_1 + 0 = 6$$

$$x_1 = 6$$

$$x_1 + 4x_2 = 12$$

$$\text{when } x_1 = 0$$

$$0 + 4x_2 = 12$$

$$x_2 = 3$$

$$\text{when } x_2 = 0$$

$$x_1 + 4(0) = 12$$

$$x_1 = 12$$

from the graph ABCD is a feasible region.

$$\min Z = 3x_1 + 2x_2$$

$$A(12, 0) \quad B(4, 2) \quad C(1, 5) \quad D(0, 10)$$

$$Z(A) = 3(12) + 2(0) = 36$$

$$Z(B) = 3(4) + 2(2) = 16$$

$$Z(C) = 3(1) + 2(5) = 13$$

$$Z(D) = 3(0) + 2(10) = 20$$

$$Z(\text{optimum}) = 13$$

Corner point C: so $x_1 = 1, x_2 = 5$

6) Mixed Constraints

Solve the LPP by using graphical method

$$\min Z = 20x_1 + 10x_2$$

S.P.C

$$x_1 + 2x_2 \leq 40$$

$$3x_1 + x_2 \geq 30$$

$$4x_1 + 3x_2 \geq 60$$

Conversion of inequalities into Equalities, then obtained Equations are.

$$x_1 + 2x_2 = 40 \quad - \text{Eq 1}$$

$$3x_1 + x_2 = 30 \quad - \text{Eq 2}$$

$$4x_1 + 3x_2 = 60 \quad - \text{Eq 3}$$

from Equation -1

$$x_1 + 2x_2 = 40$$

$$\text{when } x_1 = 0$$

$$0 + 2x_2 = 40$$

$$x_2 = \frac{40}{2}$$

x_1	0	40
x_2	20	0

$$\text{when } x_2 = 0$$

$$x_1 + 2(0) = 40$$

$$x_1 = 40$$

from Equation ①

$$3x_1 + x_2 = 30$$

x_1	0	10
x_2	30	0

when $x_1 = 0$

$$3(0) + x_2 = 30$$

$$x_2 = 30$$

when $x_2 = 0$

$$3x_1 + 0 = 30$$

$$3x_1 = 30$$

$$x_1 = 10$$

from Equation ②

$$4x_1 + 3x_2 = 60$$

x_1	0	15
x_2	20	0

when $x_2 = 0$

$$4x_1 + 3(0) = 60$$

$$4x_1 = 60$$

$$x_1 = 60/4$$

$$x_1 = 15$$

when $x_1 = 0$

$$4(0) + 3x_2 = 60$$

$$4x_1 + 3x_2 = 60$$

$$4(0) + 3x_2 = 60$$

$$3x_2 = 60$$

$$x_2 = 20$$

$$\Rightarrow \max Z = 100x_1 + 60x_2$$

S.t.c

$$5x_1 + 10x_2 \leq 50$$

$$8x_1 + 2x_2 \geq 16$$

$$3x_1 - 2x_2 \geq 6$$

Conversion of Equations from inequalities into Equalities.

then obtained Equations are.

$$5x_1 + 10x_2 = 50 \quad - (1)$$

$$8x_1 + 2x_2 = 16 \quad - (2)$$

$$3x_1 - 2x_2 = 6 \quad - (3)$$

from Eq (1)

$$5x_1 + 10x_2 = 50$$

When $x_1 = 0$.

$$5(0) + 10x_2 = 50$$

$$10x_2 = 50$$

$$x_2 = 50/10$$

$$x_2 = 5$$

x_1	0	10
x_2	5	0

When $x_2 = 0$.

$$5x_1 + 10x_2 = 50$$

$$5x_1 + 10(0) = 50$$

$$5x_1 = 50$$

$$x_1 = 50/5$$

$$x_1 = 10$$

from Eq (2)

$$8x_1 + 2x_2 = 16$$

When $x_1 = 0$

$$8(0) + 2x_2 = 16$$

$$2x_2 = 16$$

$$x_2 = 16/2$$

$$x_2 = 8$$

x_1	0	2
x_2	8	0

When $x_2 = 0$

$$8x_1 + 2(0) = 16$$

$$8x_1 = 16$$

$$x_1 = 16/8$$

$$x_1 = 2$$

from Eq ③

$$3x_1 - 2x_2 = 6$$

When $x_1 = 0$

$$3(0) - 2x_2 = 6$$

$$-2x_2 = 6$$

$$x_2 = 6/2$$

$$x_2 = -3$$

x_1	0	2
x_2	-3	0

When $x_2 = 0$

$$3x_1 - 2(0) = 6$$

$$3x_1 = 6$$

$$x_1 = 6/3$$

$$x_1 = 2$$

from the graph ABC is a feasible region

A(2,0) B(10,6) C(4,3)

$$\text{max } Z = 100x_1 + 60x_2$$

$$Z(A) = 100(2) + 60(0) = 200$$

$$Z(B) = 100(10) + 60(6) = 1600$$

$$Z(C) = 100(4) + 60(3) = 580$$

$$Z(\text{max}) = 1600$$

$$Z(\text{optimum}) = 1600$$

Corner point B ($x_1 = 10, x_2 = 6$)

Simplex Method

The concept of simplex method is similar to the graphical method. In graphical method extreme points of the feasible solution space are examined to search for optimal solution at one of them.

for LPP with several variable $n > 0$ i.e. $x_1, x_2, \dots, x_n$ we may not be able to graph the feasible region thus we need a more efficient method to suggest an optimal solution for such problems called the simplex method. The method was developed by G.B. Dantzig.

The simplex is an important term in mathematics that represents an object in 'n'-dimensional space connecting " $n+1$ " points. In one dimension a simplex is line segment connecting two points. In two dimensional it is a triangle formed by joining 3 points. In three dimensional it is four sided pyramid having four points.

The simplex method examines the extreme points in a systematic manner, repeating the same set of steps of the algorithm until an optimal solution is reached. It is for this reason that it is also called the iterative method.

Additional Variables :-

Additional variables are added in given LPP to convert into the standard form for the following reasons.

- * These variables allow us to convert inequalities into equalities, thereby converting the given LP problem into a form that is amenable to algebraic solutions.
- * These variables permit us to make a more comprehensive economic interpretation of a final solution.
- * Help us to get an initial feasible solution represented by the columns of Identity Matrix.

Type of Additional variables :-

a) Slack variables (s) :-

A slack variable represents unused resource, either in the form of time on a machine, labour hours, money, warehouses space or any number of such resource in various business problem. Since these variables yield on profit there such variables are added to the original objective function with zero co-efficient.

b) surplus variables :-

A surplus variable represents amount by which solution value exceeds a resource. These variables are also called Negative slack variables. Surplus variables like a slack variables carry on zero co-efficient in the objective function.

c) Artificial variable (A) :-

Artificial variables having no meaning in a physical sense and are only used as a tool for generating an initial LPP solution. Artificial variables are added to the original objective function with $-M$ co-efficient (max z) and $+M$ co-efficient (min z).

Simplex Algorithm

step ① : Formulation of LPP.

To formulate the LPP follow the below given rules.

- Formulate the mathematical model of the given LPP
- check whether all the b_i ($i = 1, 2, 3, \dots, n$) values are positive i.e. $b_i > 0$

c) -100 additional variables
 constraint to convert into equalities
 [OR]

convert the inequality constraints into equalities by adding additional variables to the left hand side of each constraint and assign co-efficients to those in objective function.

Types of constraints

Types of variables

1. less than or equal to

-1 slack variables is added

2. $\leq$

2) Greater than or equal to

-1 surplus variables subtracted

Step ③ :- $\geq$ set up the initial solution

Write down the co-efficients of all the variables in the LP model in the tabular form as shown below.

C_B	B	C_j $b^0 = x_B$	C_1	C_2	C_3	$\dots$	C_n	0	0	0	$\dots$	0
			x_1	x_2	x_3	$\dots$	x_n	s_1	s_2	s_3	$\dots$	s_n
(a) C_{B1}	s_1	b_1	a_{11}	a_{12}	a_{13}	$\dots$	a_{1n}	1	0	0	$\dots$	0
(b) C_{B2}	s_2	b_2	a_{21}	a_{22}	a_{23}	$\dots$	a_{2n}	0	1	0	$\dots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(m) C_{Bm}	s_m	b_m	a_{m1}	a_{m2}	a_{m3}	$\dots$	a_{mn}	0	0	0	$\dots$	1
$Z_j = \sum C_B x_j$ $C_j - Z_j$			0	0	0	$\dots$	0	0	0	0	$\dots$	0
			C_1	C_2	C_3	$\dots$	C_n	0	0	0	$\dots$	0

Where C_B = co-efficient of basic variable

B = Basic variable

b_i = value of basic variable $x_j = 0$

Calculate the $c_j - z_j$ value for all non-basic variables & examine the values of $c_j - c_j$ as follows
 a) If all $c_j - c_j \leq 0$ then the basic feasible solution is optimal.

b) If at least one $c_j - z_j \geq 0$ then it indicates that an improvement in the value of objective function z is possible.

Step 4:- Test for feasibility (variable to leave the basis) Select an entering variable to take the basic variable by using the following steps.

a) Select a column which has largest positive $c_j - z_j$ value and called as a key column

b) calculate minimum ratio for each row.

$$MR = \frac{b}{k_{cc}} = \frac{b}{\text{key column element}}$$

c) Select a row which has minimum positive value of MR. and called as key row

d) Select an intersect element of key column and key row called as key element

e) Select the leaving element (basic variable) in key row and replace it by an entering element (decision variable) from key column

Step 5:-

find the new solution.

The new solution can be calculated based on rows and the new values can be calculated for "b" and x_j and s_i columns as follows.

Key row changes:-

a) If the key element is one then the key row remains in the new simplex table.

b) If the key element is other than one then divide each key row element by the key element that is.

$$\text{key row changes} = \frac{\text{key row element}}{\text{key element}}$$

Other row changes (ORC):-

a) ORC = (old row element in that row) - (corresponding key row element) $\times$ fixed ratio.

$$\text{fixed ratio} = \frac{\text{key column element}}{\text{key element}}$$

b) formulate new simplex table by using new values

Step 6:-

Repeat the procedure.

Goto step 3 and repeat the procedure until all $c_j - z_j$ values are either negative or zero that is $c_j - z_j \leq 0$.

30/6/22 Problems on Simplex method

Optimality Condition
 for maximisation
 all $C_j - Z_j \leq 0$
 for minimisation
 all $C_j - Z_j \geq 0$

Solve the following LPP by using simplex method

$$\max Z = 12x_1 + 16x_2$$

$$\text{S.P.C} \\ = 10x_1 + 20x_2 \leq 120$$

$$8x_1 + 8x_2 \leq 80$$

$$\text{N.N.R} \\ = x_1, x_2 \geq 0$$

$$12x_1 + 16x_2 + 0s_1 + 0s_2$$

Given problem is maximisation type & given constraints is ≤ 0 so we introduce slack variable. We add slack variables to convert inequalities into Equalities.

$$12x_1 + 16x_2 + 0s_1 + 0s_2$$

S.P.C

$$10x_1 + 20x_2 + s_1 = 120$$

$$8x_1 + 8x_2 + s_2 = 80$$

N.N.R

$$x_1, x_2, s_1, s_2 \geq 0$$

Initial Simplex Table.

CBI	C_j	12	16	0	0	Sol	Ratio
	BV	x_1	x_2	s_1	s_2		
0	s_1	10	20	1	0	120	$\frac{120}{20} = 6$
0	s_2	8	8	0	1	80	$\frac{80}{8} = 10$
	Z_j	0	0	0	0	0	
	$C_j - Z_j$	12	16	0	0		

CBI = Coefficient of Basic Variable.

C_j = Co-efficient of objective function.

BV = Basic Variables.

Sol = Solution

Ratio = $\frac{\text{Solution}}{\text{Corresponding Column Element}}$

Key Column Element = $\frac{\text{highest Value of } (C_j - Z_j)}{\text{Corresponding column.}}$

Key Row Element = lowest Value. of Ratio
Corresponding Row.

Key Element $\rightarrow$ same value in
both key column & key
row

- * Select maximum value in $(C_j - Z_j)$ i.e., 16, the corresponding column is key column.
- * Select the least positive value in ratio column i.e., 6 that value corresponding row is key row.
- * Corresponding key row & key column intersection element is key element.

Values of Z_j

$$(0 \times 10) + (0 \times 8) = 0$$

$$(0 \times 20) + (0 \times 8) = 0$$

$$(0 \times 1) + (0 \times 6) = 0$$

$$(0 \times 6) + (0 \times 1) = 0$$

$$(0 \times 120) + (0 \times 80) = 0$$

So our optimality condition is All $C_j - Z_j \neq 0$ so we can't reach the optimality condition then go to iteration - I.

For Iteration - I x_2 is the Entering Variable & s_1 is the leaving Variable.

Iteration - I.

CB	C_j	12	16	0	0		
	BV	x_1	x_2	s_1	s_2	Sol	Ratio
16	x_2	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{20}$	0	6	$\frac{6}{\frac{1}{2}} = 12$
0	s_2	4	0	$-\frac{2}{5}$	1	32	$\frac{32}{4} = 8$
	Z_j	8	16	$\frac{4}{5}$	0	96	
	$(C_j - Z_j)$	4	0	$-\frac{4}{5}$	0		

* First we write G values and Basic Variable values. There is no change in G & BV just copy from the 1st table.

* In previous table we found s_1 is the leaving variable. x_2 is the entering variable. So we write x_2 instead of s_1 because there is a key Element.

* In order to find new values for key Element row divide all Elements by key Elements i.e., $\frac{10}{20}, \frac{20}{20} = \frac{1}{2}$

$$\frac{20}{20} = 1$$

$$\frac{6}{20} = 0$$

$$\frac{120}{20} = 6$$

other than key Row New Value

Old Value - $\frac{\text{Corresponding Key Column Element} \times \text{Key Row Value}}{\text{Key Element}}$

$$1^{\text{st}} \quad NV = OV - \frac{CKCE \times CKRE}{KE}$$

$$= 8 - \frac{8 \times 10}{20} = 8 - 4 = 4$$

2nd

$$NV = 6 - \frac{8 \times 20}{20} = 0$$

$$NV = 0 - \frac{8 \times 1}{20} = -\frac{2}{5}$$

$$NU = 80 - \frac{16}{20} = 32$$

Values of Z_j

$$(16 \times \frac{1}{2}) + (0 \times 4) = 8$$

$$(16 \times 1) + (0 \times 0) = 16$$

$$(16 \times \frac{1}{20}) + (0 \times -\frac{2}{5}) = \frac{4}{5}$$

$$(16 \times 0) + (0 \times 1) = 0$$

$$(16 \times 6) + (0 \times 32) = 96$$

optimality is not reached in Iteration-I
so go to next Iteration.

$$\text{Ratio} = \frac{6}{\frac{1}{2}} = 12$$

$$\frac{32}{4} = 8$$

x_1 is Entering variable

s_2 is leaving variable.

because key element is existed

Iteration - II

CBI	C_j	12	16	0	0		Ratio
	BV	x_1	x_2	s_1	s_2		
16	x_2	0	1	$\frac{1}{10}$	$-\frac{1}{5}$	2	
12	x_1	1	0	$-\frac{1}{10}$	$\frac{1}{4}$	8	
	Z_j	12	16	$\frac{2}{5}$	1	128	

$$\frac{A}{4} = 1$$

$$0/4 = 0$$

$$\frac{-2/5}{4} = -1/10$$

$$y_4$$

$$\frac{32}{4} = 8$$

$$NU = DV - \frac{CKCE \times CKRE}{KEU}$$

$$3 = V_2 - \frac{V_2 \times 4}{4} = 0$$

$$= 1 - \frac{V_2 \times 0}{4} = 1$$

$$0 = -1/20 = \frac{V_2 \times (-20)}{4} = 1/10$$

$$= 0 - \frac{V_2 \times 1}{4} = -1/8$$

$$= 6 - \frac{V_2 \times 32}{4} = 2$$

Value of Z_j

$$(16 \times 0) + (12 \times 0) = 12$$

$$(16 \times 1) + (12 \times 6) = 16$$

$$(16 \times \frac{1}{10}) + (12 \times -\frac{1}{10}) = 2/5$$

$$(16 \times -1/8) + (12 \times 1/4) = 1$$

$$(16 \times 2) + (12 \times 8) = 128$$

$$\text{② } \max Z = 36x_1 + 8x_2$$

$$s.t. 5x_1 + 10x_2 \leq 60$$

$$4x_1 + 4x_2 \leq 40$$

$$x_1, x_2 \geq 0$$

Sol Max $Z = 6x_1 + 8x_2$

Sub to Constraints

$$5x_1 + 10x_2 + S_1 = 60$$

$$4x_1 + 4x_2 + S_2 = 40$$

$$x_1, x_2, S_1, S_2 \geq 0$$

Initial simplex table.

CBJ	Cj	6	8	0	0	Sol	Ratio
	BV	x_1	x_2	S_1	S_2		
0	S_1	5	10	1	0	60	$\frac{60}{10} = 6 \leftarrow$
0	S_2	4	4	0	1	40	$\frac{40}{4} = 10$
	Z_j	0	0	0	0		
	$C_j - Z_j$	6	8	0	0		

$C_j - Z_j$ is not ≤ 0 so we need to

go for iteration - 1

Entering Variable: x_2

Leaving Variable: S_1

Iteration - 1

CBJ	Cj	6	8	0	0	Sol	Ratio
	BV	x_1	x_2	S_1	S_2		
8	x_2	$\frac{1}{2}$	1	$\frac{1}{10}$	0	6	$\frac{6}{\frac{1}{2}} = 12$
0	S_2	2	0	$-\frac{2}{5}$	1	16	$\frac{16}{2} = 8 \leftarrow$
	Z_j	4	8	$\frac{4}{5}$	0	48	
	$C_j - Z_j$	2	0	$-\frac{4}{5}$	0		

Sol Max $z = 6x_1 + 8x_2$

Sub to Constraints

$$5x_1 + 10x_2 + S_1 = 60$$

$$4x_1 + 4x_2 + S_2 = 40$$

$$x_1, x_2, S_1, S_2 \geq 0$$

Initial simplex table.

CBS	Cj	6	8	0	0	sol	Ratio
	BV	x_1	x_2	S_1	S_2		
0	S_1	5	10	1	0	60	$\frac{60}{10} = 6 \leftarrow$
0	S_2	4	4	0	1	40	$\frac{40}{4} = 10$
	Z_j	0	0	0	0		
	$C_j - Z_j$	6	8	0	0		

$C_j - Z_j$ is not ≤ 0 so we need to

go for iteration - 1
 Entering Variable:
 x_2

Leaving Variable:
 S_1

Iteration - 1.

CBS	Cj	6	8	0	0	sol	Ratio
	BV	x_1	x_2	S_1	S_2		
8	x_2	$\frac{1}{2}$	1	$\frac{1}{10}$	0	6	$\frac{6}{1/2} = 12$
0	S_2	2	0	$-\frac{2}{5}$	1	16	$\frac{16}{2} = 8 \leftarrow$
	Z_j	4	8	$\frac{4}{5}$	0	48	

$$NV = OV - \frac{CKCE \times CKRE}{KE}$$

$$= 4 - \frac{4 \times 5}{10}$$

$$= 4 - \frac{4 \times 10}{10}$$

$$= 4 - \frac{4}{10}$$

$$= 1 - \frac{0}{10}$$

$$= 40 - \frac{60 \times 4}{10} = 16$$

for all $C_j - Z_j$ is not ≤ 0

$\rightarrow x_1$ is Entering variable.

s_2 is leaving variable.

Iteration - 2

(BV)	C_j	6	8	0	0		
	BV	x_1	x_2	s_1	s_2	sol	Ratio
8	x_2	0	1	$1/5$	$-1/4$	2	
6	x_1	1	0	$-1/5$	$1/2$	8	
	Z_j	6	8	$2/5$	1	64	
	$C_j - Z_j$	0	0	$-2/5$	-1		

$$\text{So } x_1 = 3, x_2 = 8$$

$$Z(\text{optimum}) = 64$$

Solve the LP by using Simplex Method
two constraints & 3 variables.

$$\max Z = 10x_1 + 15x_2 + 20x_3$$

s.p.c

$$= 2x_1 + 4x_2 + 6x_3 \leq 24$$

$$3x_1 + 9x_2 + 6x_3 \leq 30$$

$$x_1, x_2, x_3 \geq 0$$

Sol

$$\max Z = 10x_1 + 15x_2 + 20x_3 + 0s_1 + 0s_2$$

s.p.c

$$2x_1 + 4x_2 + 6x_3 + s_1 = 24$$

$$3x_1 + 9x_2 + 6x_3 + s_2 = 30$$

N.N.R

$$x_1, x_2, x_3, s_1, s_2 \geq 0$$

Initial Simplex table

C.B.F	C_j	10	15	20	0	0		
	B.V	x_1	x_2	x_3	s_1	s_2	sol	Ratio
0	s_1	2	4	6	1	0	24	4 ←
0	s_2	3	9	6	0	1	30	5
	Z_j	0	0	0	0	0		
	$C_j - Z_j$	10	15	20 ↑	0	0		

for all $C_j - Z_j$ is not ≤ 0 then we go to next Iteration - 1

x_3 is Entering Variable.
 s_1 is leaving Variable.

Iteration - 1

CBS	C_j	10	15	20	0	0		
	BV	x_1	x_2	x_3	s_1	s_2	sol	Ratio
20	x_3	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{1}{6}$	0	4	$\frac{4 \times 1}{1} = 4$
0	s_2	1	5	0	-1	1	6	6
	Z_j	$\frac{20}{3}$	$\frac{40}{3}$	20	$\frac{10}{3}$	0		
	$C_j - Z_j$	$\frac{10}{3}$	$\frac{5}{3}$	0	$-\frac{10}{3}$	0		

for all $C_j - Z_j$ is not ≤ 0 then we go

Iteration - 2

x_1 is entering Variable.
 s_2 is leaving Variable.

Iteration - 2

CBS	C_j	10	15	20	0	0		
	BV	x_1	x_2	x_3	s_1	s_2	sol	Ratio
20	x_3	0	-1	1	$\frac{1}{6}$	$-\frac{1}{3}$	2	$\frac{2 \times 1}{1} = 2$
10	x_1	1	5	0	-1	1	6	
	Z_j	10	30	20	0	$\frac{10}{3}$	100	
	$C_j - Z_j$	0	-15	0	0	$-\frac{10}{3}$		

$$C_j - Z_j \leq 0 \quad \text{so}$$

$$x_1 = 6, x_2 = 0, x_3 = 3.$$

$$Z(\text{optimum}) = 100$$

Solve the following LPP by using simplex method

$$\max Z = 20x_1 + 20x_2 \quad 40x_1 + 35x_2$$

S.P.C

$$2x_1 + 4x_2 \leq 60$$

$$3x_1 + 3x_2 \leq 96$$

N.N.R

$$x_1, x_2 \geq 0$$

So) Objective function

$$40x_1 + 35x_2 + 0s_1 + 0s_2$$

S.P.C

$$2x_1 + 4x_2 + s_1 = 60$$

$$3x_1 + 3x_2 + s_2 = 96$$

N.N.R

$$x_1, x_2, s_1, s_2 \geq 0$$

Initial simplex method

C.B.I	C_j	40	35	0	0		
	B.V	x_1	x_2	s_1	s_2	sol	Ratio
0	s_1	2	4	1	0	60	$\frac{60}{2} = 30 \leftarrow$
0	s_2	3	3	0	1	96	$\frac{96}{3} = 32$
	Z_j	0	0	0	0	0	
	$C_j - Z_j$	40	35	0	0		

for all $C_j - Z_j$ is not ≤ 0 so we go

Iteration - 2

x_1 is entering variable.

S_1 is leaving variable.

Iteration - 2

CBS	C_j	40	35	0	0		
	BU	x_1	x_2	S_1	S_2	sol	Ratio
40	x_1	1	2	$\frac{1}{2}$	0	20	
0	S_2	0	-3	$-\frac{3}{2}$	1	6	
	Z_j	40	80	20	0	1200	
	$C_j - Z_j$	0	-45	-20	0		

$$x_1 = 20, x_2 = 0, S_1 = 0$$

$$Z(\text{optimum}) = 1200$$

Three Variables & three Constraints.

$$\text{max } Z = 3x_1 + 5x_2 + 4x_3$$

S.P.C

$$2x_1 + 8x_2 \leq 8$$

$$2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

CBS		3	5	4	0	0	0		
	BV	x_1	x_2	x_3	s_1	s_2	s_3	Sol	Ratio
0	s_1	2	3	0	1	0	0	8	$18/3$
0	s_2	0	2	5	0	1	0	10	5
0	s_3	3	2	4	0	0	1	15	$\frac{15}{2}$
	Z_j	0	0	0	0	0	0	0	
	$C_j - Z_j$	3	5	4	0	0	0		

for all $C_j - Z_j \neq 0$ so we go to Iteration-1

x_2 is Entering Variable.

s_1 is leaving Variable.

CBS		3	5	4	0	0	0		
	BV	x_1	x_2	x_3	s_1	s_2	s_3	Sol	Ratio
5	x_2	$2/3$	1	0	$1/3$	0	0	$8/3$	-
0	s_2	$-4/3$	0	5	$-2/3$	1	0	$\frac{14}{3}$	$\frac{14}{15}$
0	s_3	$5/3$	0	4	$-2/3$	0	1	$\frac{29}{3}$	$\frac{29}{12}$
	Z_j	$\frac{10}{3}$	5	0	$\frac{5}{3}$	0	0	$\frac{40}{3}$	
	$C_j - Z_j$	$-1/3$	0	4	$-5/3$	0	0		

for all $C_j - Z_j \neq 0$ so we go to iteration-2

x_3 is Entering Variable.

s_2 is Leaving Variable.

CBI	C_j	3	5	4	0	0	0	sol	Ratio
BV	x_1	x_2	x_3	s_1	s_2	s_3			
5	x_2	$\frac{2}{5}$	1	0	$\frac{1}{2}$	0	0	$\frac{8}{3}$	4
4	x_3	$-\frac{4}{5}$	0	1	$-\frac{2}{15}$	$\frac{1}{5}$	0	$\frac{14}{15}$	-
0	s_3	$\frac{4}{15}$	0	0	$\frac{2}{15}$	$-\frac{4}{5}$	1	$\frac{89}{15}$	$\frac{89}{41}$
	Z_j	$\frac{34}{15}$	5	4	$\frac{12}{15}$	$\frac{4}{5}$	0	$\frac{256}{15}$	
	$C_j - Z_j$	$\frac{11}{15}$	0	0	$-\frac{13}{15}$	$-\frac{4}{5}$	0		

CBI	C_j	3	5	4	0	0	0	sol	Ratio
BV	x_1	x_2	x_3	s_1	s_2	s_3			
5	x_2	1	0	$\frac{1}{2}$	0	0			
4	x_3	0	1	0	$\frac{1}{2}$	0			
3	x_1	0	0	0	0	$\frac{1}{2}$			

solve the LP by using simplex method

$$\max Z = 4x_1 + 3x_2$$

$$4x_1 + 3x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4$$

S.P.C

$$2x_1 + x_2 \leq 1000$$

$$2x_1 + x_2 + S_1 = 1000$$

$$x_1 + x_2 \leq 800$$

$$x_1 + x_2 + S_2 = 800$$

$$x_1 \leq 400$$

$$x_1 + S_3 = 400$$

$$x_2 \leq 700$$

$$x_2 + S_4 = 700$$

CBP	C_j	4	3	0	0	0	0	Sol	Ratio
BV		x_1	x_2	S_1	S_2	S_3	S_4		
0	S_1	2	1	1	0	0	0	1000	500
0	S_2	1	1	0	1	0	0	800	800
0	S_3	1	0	0	0	1	0	400	400 ←
0	S_4	0	1	0	0	0	1	700	-
	Z_j	0	0	0	0	0	0		
	$C_j - Z_j$	4	3	0	0	0	0		

for all $C_j - Z_j \geq 0$ S_3 is L.V. x_1 is E.V.

CBP	C_j	4	3	0	0	0	0	Sol	Ratio
BV		x_1	x_2	S_1	S_2	S_3	S_4		
0	S_1	0	1	1	0	2	0	200	200 ←
0	S_2	0	1	0	0	-1	0	400	400
4	x_1	1	0	0	0	1	0	400	-
0	S_4	0	1	0	0	0	0	700	700
	Z_j	4	0	0	0	4	0	1600	
	$C_j - Z_j$	0	3	0	0	-4	0		

for all $C_j - Z_j \leq 0$, S_1 is leaving variable.

Iteration - 2

C.B.T	C_j	4	3	0	0	0	0	Sol	Ratio
	B.V	x_1	x_2	S_1	S_2	S_3	S_4		
3	x_2	0	1	1	0	-2	0	200	-100
0	S_2	0	0	-1	0	1	0	200	200
4	x_1	1	0	0	0	1	0	400	400
0	S_4	0	0	-1	0	2	0	1400	700
	Z_j	4	3	0	0	-2	0	-1000	
	$C_j - Z_j$	0	0	-3	0	+2	0		

S_3 is Ent var
 S_2 is leav. var

for all $C_j - Z_j \leq 0$

Iteration - 3

C.B.T	C_j	4	3	0	0	0	0	Sol	Ratio
	B.V	x_1	x_2	S_1	S_2	S_3	S_4		
3	x_2	0	1	-1	0	0	0	600	
0	S_3	0	0	-1	0	1	0	200	
4	x_1	1	0	1	0	0	0	200	
0	S_4	0	0	1	0	0	0	1000	
	Z_j	4	3	1	0	0	0	2600	
	$C_j - Z_j$	0	0	-1	0	0	0		

$$x_1 = 200, \quad x_2 = 600$$

$$Z(\text{optimum}) = 2600$$

Solve the LPP by using simplex method

$$\min Z = x_1 - 3x_2 + 2x_3$$

S.T.C

$$3x_1 - x_2 + 2x_3 \leq 4$$

$$-2x_1 + 4x_2 \leq 12$$

$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

NNR

$$x_1, x_2, x_3 \geq 0$$

Q.1) the given problem is minimization problem. Convert this minimization problem into maximization problem by multiplying with '-1' to the objective function. Now the objective function is

$$\min Z = \max(-Z)$$

$$\max Z = -x_1 + 3x_2 + 2x_3$$

S.T.C

$$3x_1 - x_2 + 2x_3 + s_1 = 4$$

$$-2x_1 + 4x_2 + s_2 = 12$$

$$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$$

Obj func $\max Z = -x_1 + 3x_2 + 2x_3 + 0s_1 + 0s_2 + 0s_3$

NNR

$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

Initial simplex table

CBT	C _j	-1	3	-2	0	0	0		
BV	x ₁	x ₂	x ₃	s ₁	s ₂	s ₃		sol	Ratio
0	s ₁	3	-1	2	1	0	0	7	-7
0	s ₂	-2	4	0	0	1	0	12	3/4
0	s ₃	-4	3	8	0	0	1	10	10/3
Z _j	0	0	0	0	0	0	0	0	
C _j -Z _j	-1	3	-2	0	0	0	0		

for all $C_j - Z_j$ is not ≤ 0 then
 go to iteration - I
 x_2 is entering variable
 s_2 is leaving variable.

Iteration 1

	C_j	-1	3	-2	0	0	0		
CBT	BV	x_1	x_2	x_3	s_1	s_2	s_3	sol	Ratio
0	s_1	5/2	0	2	1	1/4	0	10	4
3	x_2	-1/2	1	0	0	1/4	0	3	2
0	s_3	-5/2	0	8	0	-1	0	1	2/5
	Z_j	-3/2	3	0	0	3/4	0	9	
	$C_j - Z_j$	1/2	0	-2	0	-3/4	0		

x_1 is Entering Variable.

s_1 is leaving Variable.

	C_j	-1	3	-2	0	0	0		
CBT	BV	x_1	x_2	x_3	s_1	s_2	s_3	sol	Ratio
-1	x_1	1	0	4/5	2/5	1/10	0	4	
3	x_2	0	1	2/5	1/5	1/20	0	5	
0	s_3	0	0	10	5	-3/4	0	11	
	Z_j	-1	3	2/5	1/5	+1/20	0	11	
	$C_j - Z_j$	0	0	-8/5	-1/5	-1/20	0		

for all $C_j - Z_j \leq 0$, $\max z = 11$
 $\min z = -\max(-z)$

$$x_1 = 4, x_2 = 5$$

$z(\text{optimum}) = 11$ for maximum

for min $z(\text{optimum}) = -11$

Minimize Constraints

$$\text{max } Z = 8x_1 - 4x_2$$

$$\text{S.T.C } 4x_1 + 5x_2 \leq 20$$

$$-x_1 + 3x_2 \geq -23$$

$$x_1, x_2 \geq 0$$

Multiply Eq (2) with '-1' to get max form

Initial Simplex Table

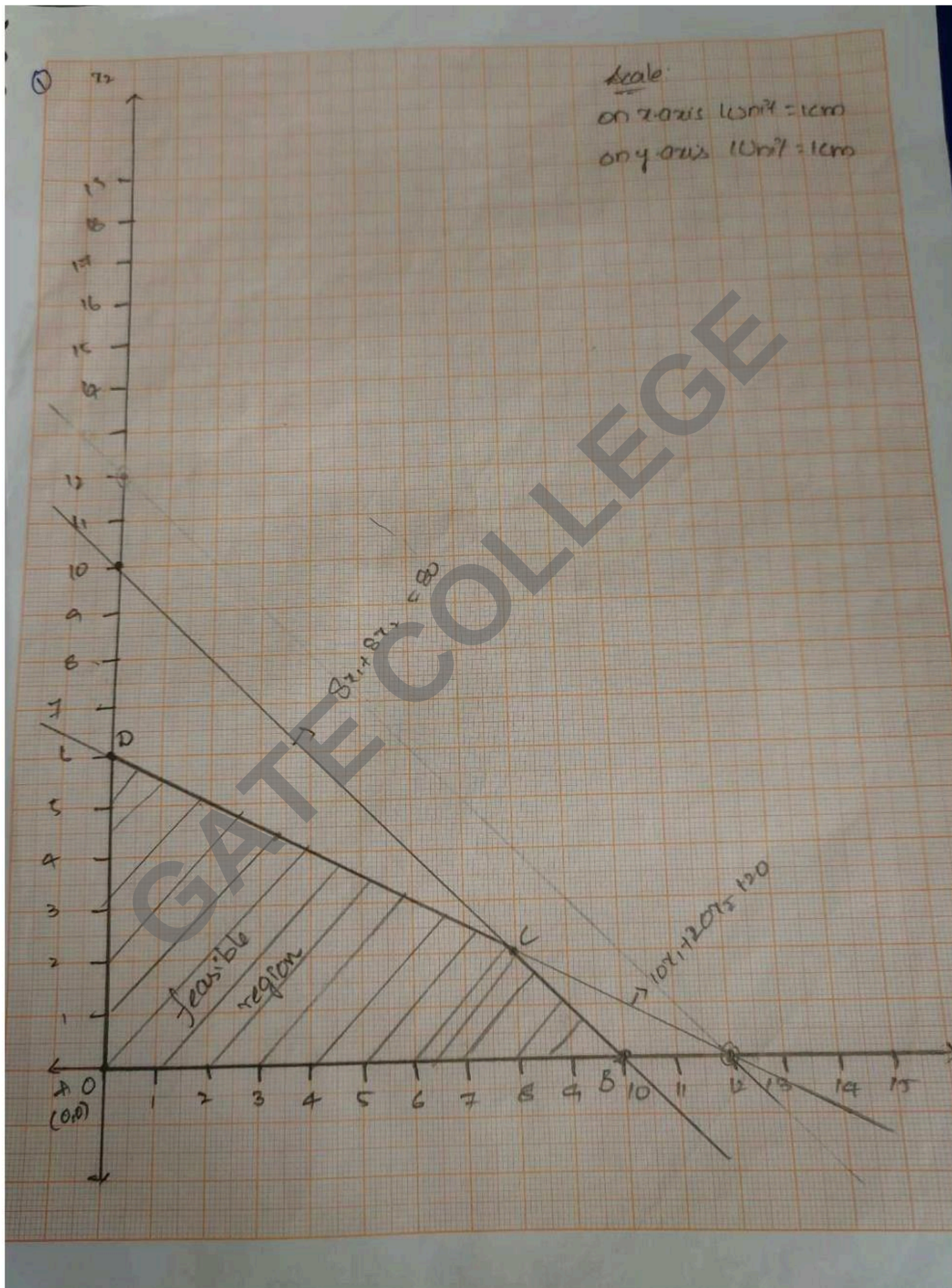
CBJ	Cj	8	-4	0	0		
BV	x ₁	x ₂	s ₁	s ₂		sol	Ratio
0	s ₁	4	5	1	0	20	5
0	s ₂	1	-3	0	1	23	23
	Z _j	0	0	0	0	0	
	C _j -Z _j	8	-4	0	0		

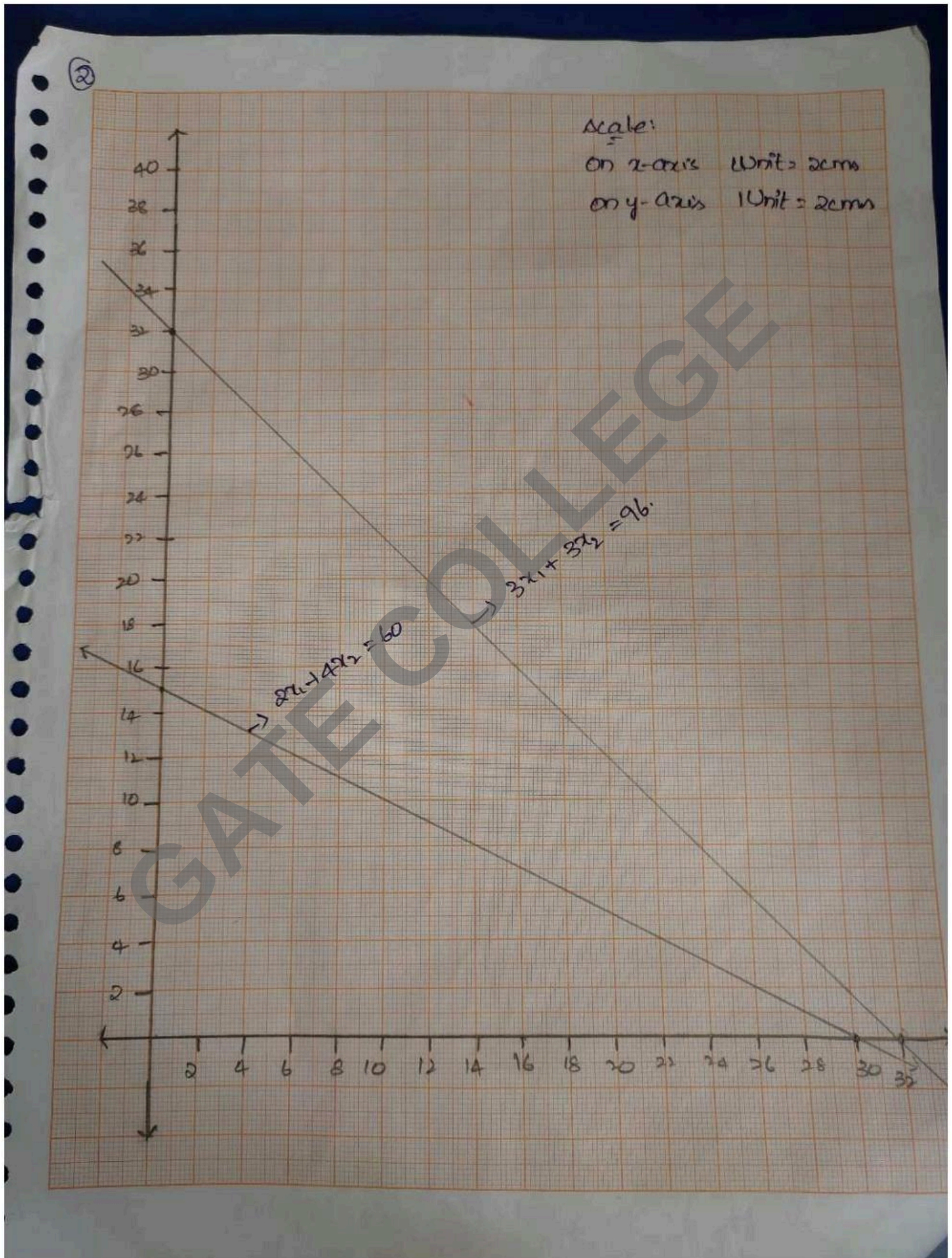
Iteration-2

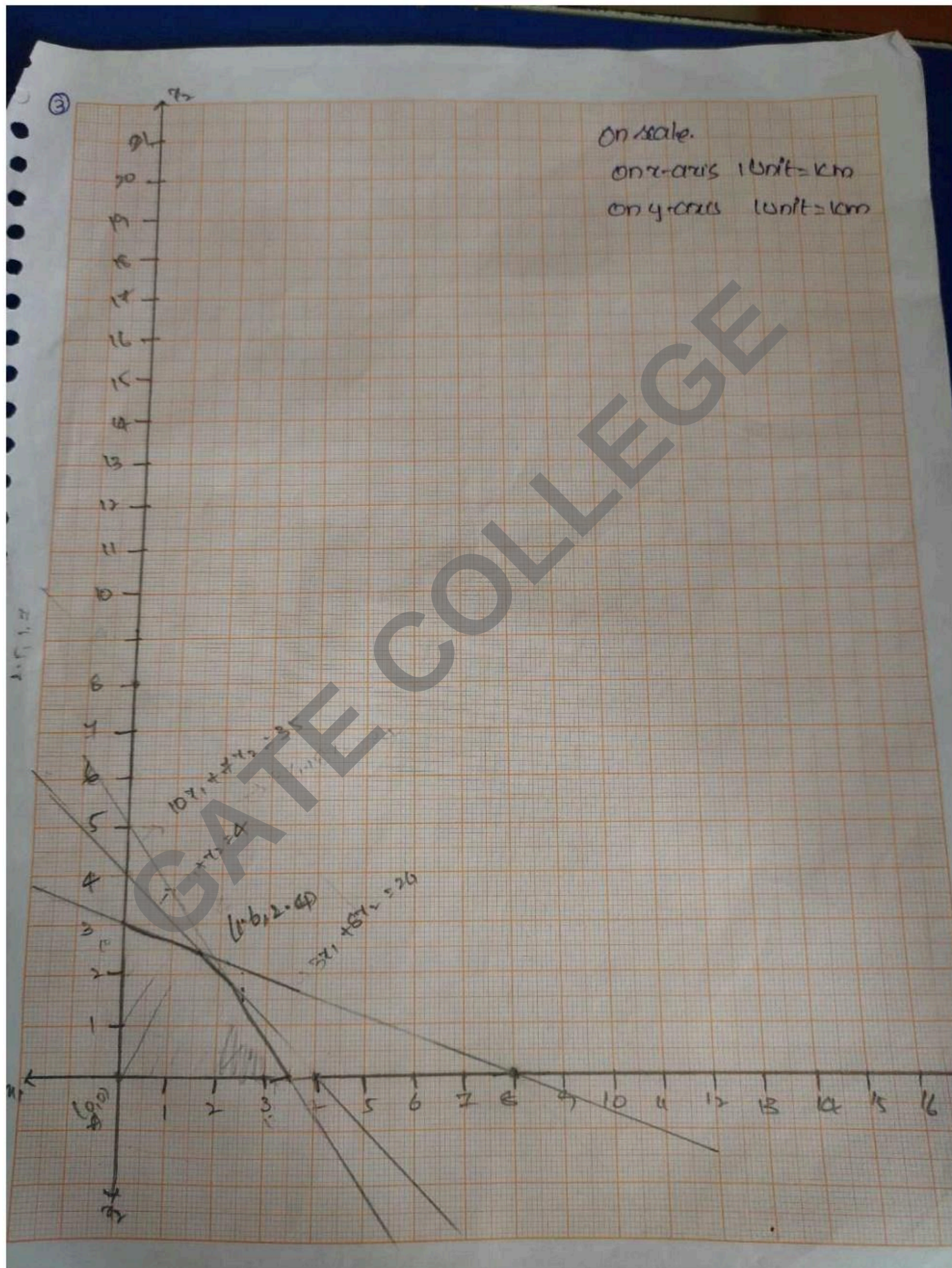
CBJ	Cj	8	-4	0	0		
BV	x ₁	x ₂	s ₁	s ₂		sol	Ratio
8	x ₁	1	5/4	1/4	0	5	
0	s ₂	0	-17/4	3/4	0	18	
	Z _j	8	10	2	0	40	
	C _j -Z _j	0	-14	-2	0		

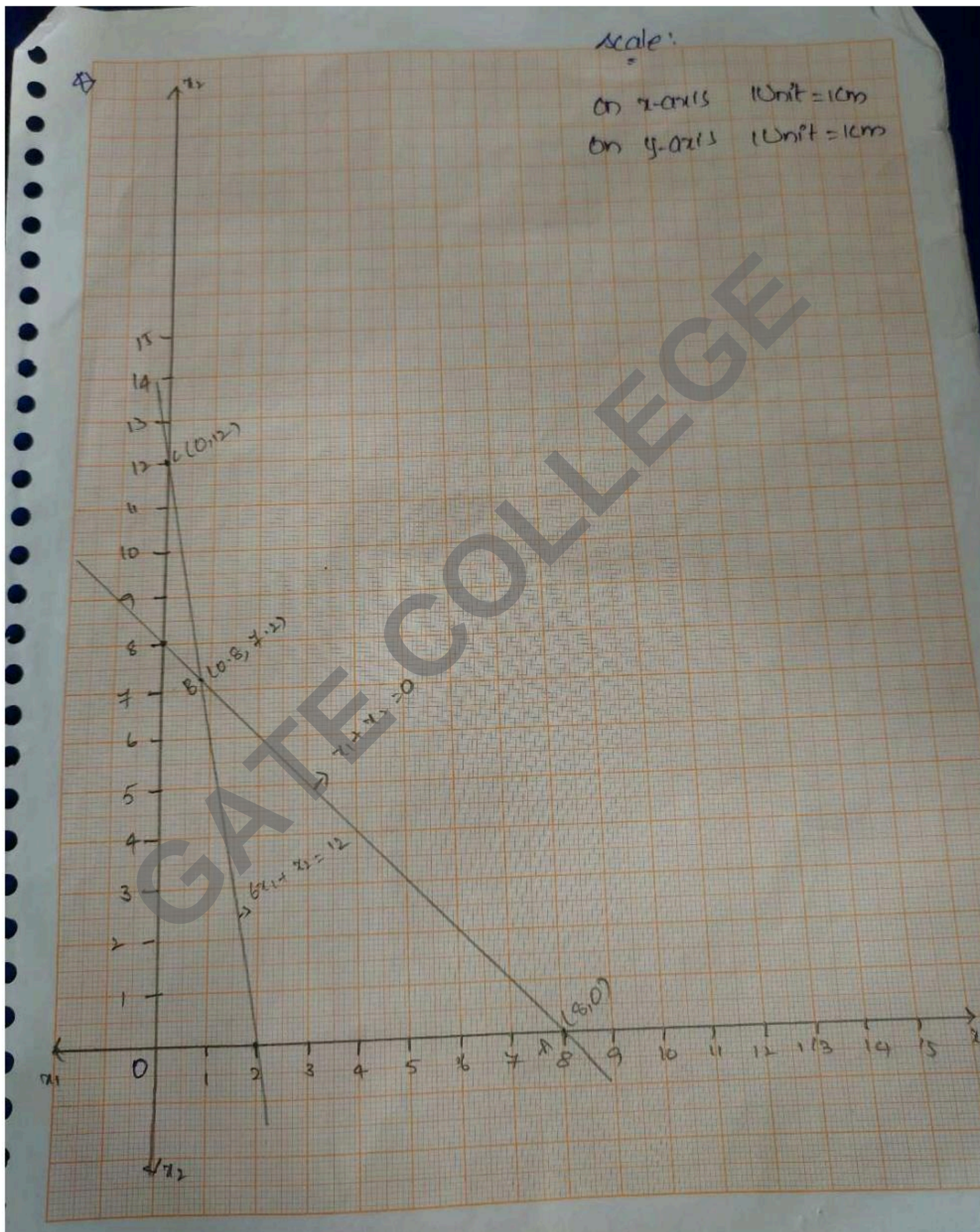
Lpp Excel Solver

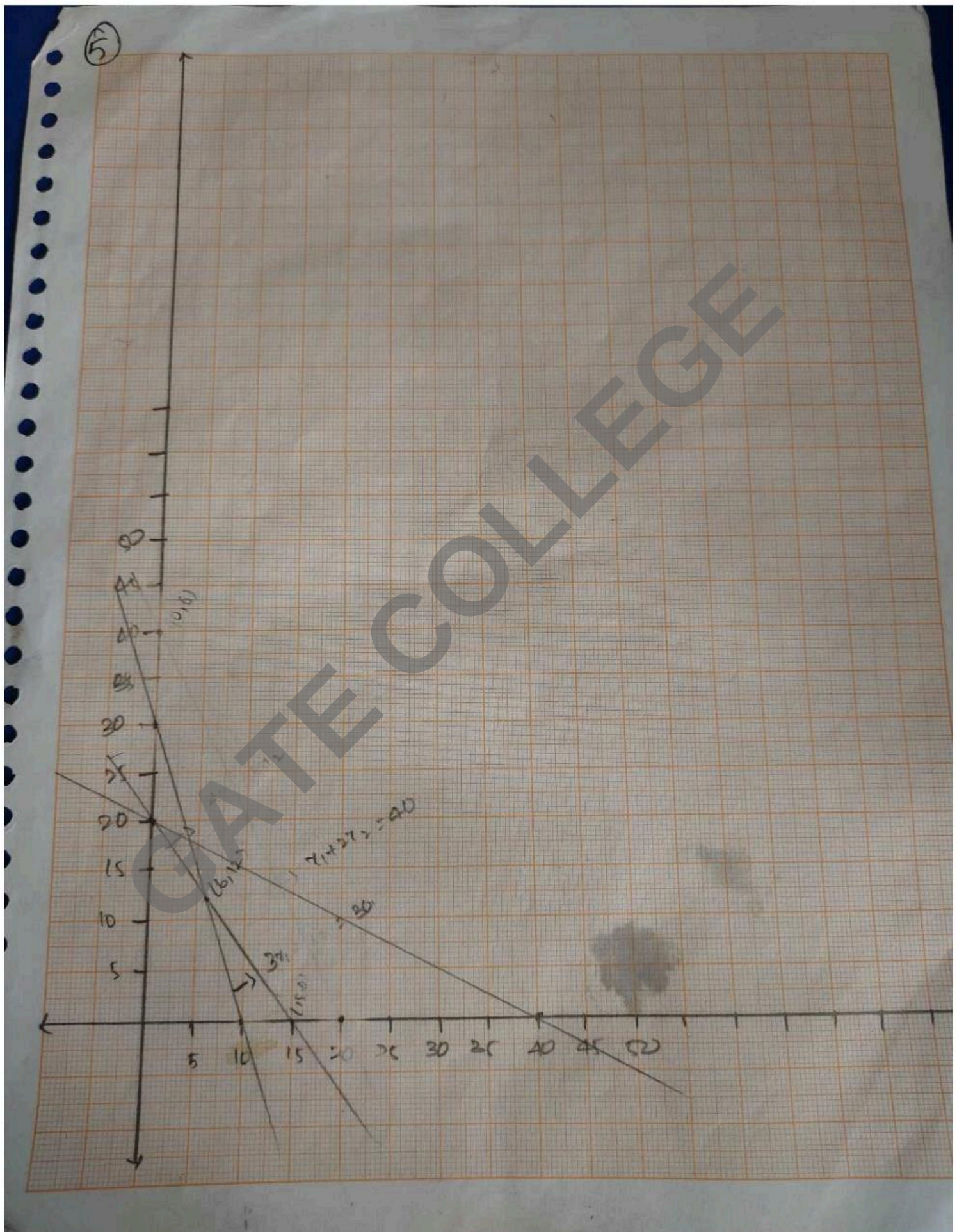
- Step ① - click Start button → goto M.S. Office → goto M.S. Excel → click ok.
- Step ② - Now Excel sheet is to be saved.
- Step ③ - To Enter the given problem of the decision Variables of objective function in the first column
- Step ④ - Enter the subjective constraints and non-negativity Restrictions in the rows of the Excel sheet.
- Step ⑤ - Goto Data Analysis pack (DAP) Excel
- Step ⑥ - click the Add on → Add the Solver.
- Step ⑦ - Click ok the Excel Solver.

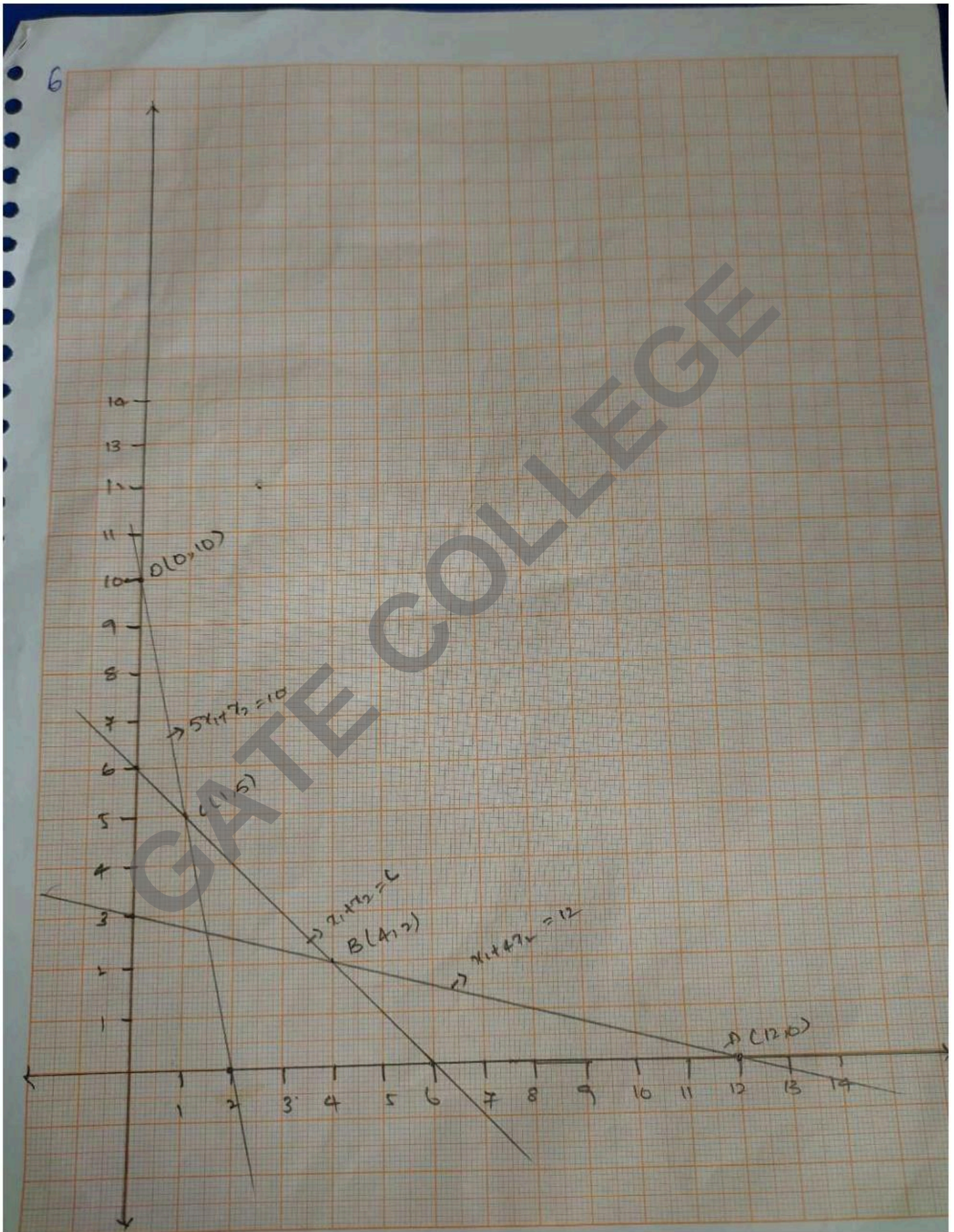


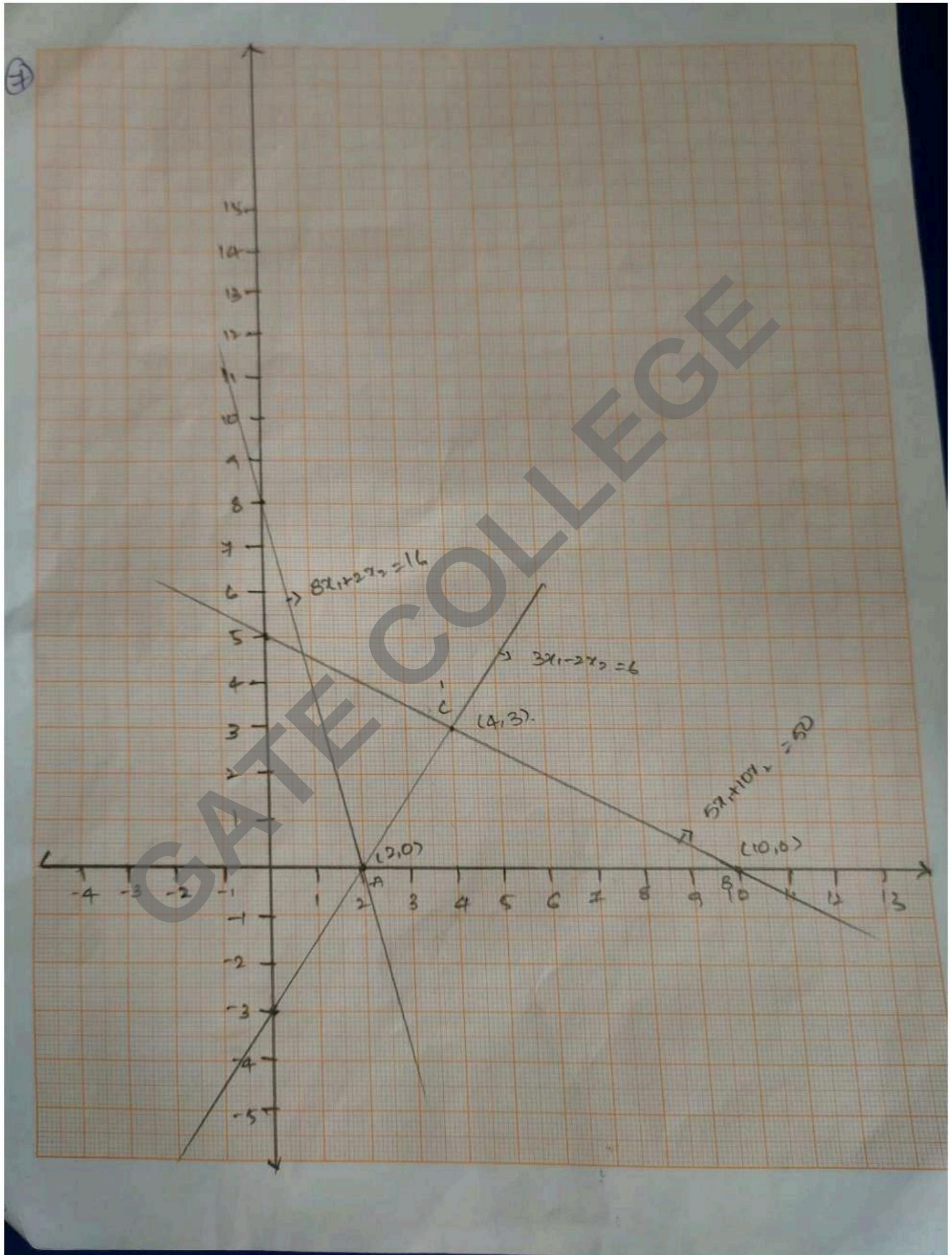












TRANSPORTATION PROBLEM DEGENERACY

Introduction:- Transportation problem is a special kind of LPP. In which goods are transported from a set of sources to set of destinations. Subject to the supply and demand of the sources and destinations respectively such that the total cost of transportation is minimised.

The transportation problem deal with the distribution of a product manufactured at 'm' Number of plants to (supply origins) 'm' Number of warehouses (Demand destinations).

There are various type of transportation models and the simplest of them was first presented by Alfred F.L. Hitchcock in the year 1949 and T.B. Dantzig in the year 1951. Several extensions of after transportation model and methods have been subsequently developed.

MEANING:-

A shipment of commodities takes place from source to destination directly is called transportation.

* The transportation problem is to transport various amounts of a single homogeneous commodity that are initially stored at various origins, to different destinations in such a way that the total transportation cost is minimised.

the same product. From these factories the product is distributed to three warehouses w_1, w_2, w_3 . Each Factory has a limited supply and each warehouse has specific demand. Each factory can transport to each warehouse but the transportation cost vary from different combination. the problem is to determine the quantity each factory should transport to each warehouse in order to minimum the total transportation cost.

MATHEMATICAL MODEL of TRANSPORTATION PROGRAMMING (Tpp):-

Let there be 'm' number of sources $S_1, S_2, S_3, \dots, S_m$ having a_i ($i=1, 2, 3, \dots, m$) units of supply respectively to be transported among 'n' number of destinations $D_1, D_2, D_3, \dots, D_n$ with b_j ($j=1, 2, 3, \dots, n$) units of demand respectively.

Let c_{ij} be the cost of shipping one unit of commodity from source 'i' to destination 'j' for each route. If x_{ij} represents number of units shipped per route from source i to destination j, the problem is to determine the transportation schedule so as to minimise the total transportation cost satisfying supply and demand conditions. Mathematically the transportation problem may be stated as follows.

Objective function:-

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

Subject to constraints:-

$$\text{Supply constraint } \sum_{j=1}^n x_{ij} = a_i; \quad i=1, 2, 3, \dots, m$$

Demand constraint:- $\sum_{i=1}^m x_{ij} = b_j; j=1, 2, 3, \dots, n$

Restrictions:- $x_{ij} \geq 0$ (Non-negative variables)

Matrix form of TPP.

		Destinations					Supply
		D_1	D_2	D_3	$\dots$	D_n	
Sources	S_1	c_{11}	c_{12}	c_{13}	$\dots$	c_{1n}	a_1
	S_2	c_{21}	c_{22}	c_{23}	$\dots$	c_{2n}	a_2
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	S_m	c_{m1}	c_{m2}	c_{m3}	$\dots$	c_{mn}	$\sum a_i = \sum b_j$
Demand		b_1	b_2	b_3	$\dots$	b_n	$\sum a_i = \sum b_j$

where m = number of sources

n = number of destinations

a_i = supply at the source;

b_j = Demand at the destination j

c_{ij} = cost of transportation per unit from source i to destination j .

Types of Transportation problem:-

The Transportation problem can be classified into two types.

1. Balanced Transportation problem.

2. Unbalanced transportation problem.

Balanced Transportation problem:-

If the sum of the supplies of all the sources is equal to the sum of the demands of all the destinations then the problem is termed as Balanced

Transportation problem $\sum a_i = \sum b_j$

		Destination			
		1	2	3	4
Source	1	3	1	7	4
	2	2	6	5	9
	3	8	3	3	2
	Demand	250	350	400	200
		Supply			
		300	400	500	

The sum of the supplies 1200 = sum of the demands 1200
Hence the transportation problem is balanced transportation problem.

Unbalanced transportation problems:-

If the sum of the supplies of all the source is not equal to the sum of the demands of all the destinations. then the problem is turned as unbalanced transportation problem i.e., $\sum a_i \neq \sum b_j$.

		Destinations		
		1	2	3
Source	1	30	50	15
	2	35	70	20
	3	20	45	60
	Demand	300	200	400
		Supply		
		300	200	500

Sum of Supplies = 1000

Sum of Demands = 900

$$\sum a_i \neq \sum b_j$$

So this problem is unbalanced transportation problem. Convert the unbalanced transportation problem into a balanced transportation problem by adding Dummy Destination to Absorb the excess supply.

the demand of the dummy destination is equal to $\sum_{i=1}^m a_i - \sum_{j=1}^n b_j$, so the cost co-efficient's in the dummy destination are assumed as zeros. If $\sum_{j=1}^n b_j > \sum_{i=1}^m a_i$,

then include a Dummy source to supply the excess demand the supply of the Dummy source is equal to $\sum_{j=1}^n b_j - \sum_{i=1}^m a_i$; the cost co-efficient's in the dummy Source are assumed as zeros.

	Destination				Supply
	1	2	3	4	
Source 1	30	50	15	0	300
Source 2	35	70	20	0	200
Source 3	20	45	60	0	500
Demand					300 200 400 100

Problems:-

- i) Convert the unbalanced transportation problem into balanced transportation.

	Destination				Supply
	1	2	3	4	
Source 1	5	12	6	10	300
Source 2	7	8	10	3	400
Source 3	9	4	9	2	300
Demand					200 300 450 250

Soln

Destination.

	1	2	3	4	Supply
Source 1	5	12	6	10	300
2	7	8	10	3	400
3	9	4	9	2	300
4	0	0	0	0	200
Demand	200	300	450	250	1200

Here supply and Demand is not same

So we add dummy row in source and add the cost to Demand.

$\therefore$ Supply = Demand.

Methods of Transportation problem:-

The solution procedure for the transportation problem consists of two phases.

1. Finding the initial basic feasible solution is the first phase.
2. Second phase involves optimization of the initial basic feasible solution which is obtained in phase one.

1. Finding the initial basic feasible solution (IBFS):-

There are five types of techniques available to find the initial basic feasible solution. the solution using these techniques may not be optimal.

1. Northwest corner cell method (NWCM)
2. Least cost cell method (LCCM)/matrix minimum method
3. Vogel's Approximation method (VAM)/penalty method.

4. Row minima method.

5. Column minima method.

Algorithm for NorthWest corner cell method:-

1. Find the minimum of the supply and Demand values with respect to the current North-west corner cell of the Cost matrix.
2. Allocate these minimum value to the current North-west corner cell and subtract these minimum from the supply and Demand values with respect to the current North-west corner cell.
3. Check whether exactly one of the row (or) column corresponding to the North-west corner cell has zero supply or demand, respectively. If so go to step-4. Otherwise go to step-5.
4. Delete that row (or) column with respect to the current North-west corner cell which has the zero supply (or) demand and goto step-6
5. Delete both the row and the column with respect to the current North-west corner cell.
6. Check whether exactly one row (or) column is left out. If yes goto step-7 otherwise goto step-1.
7. Match the supply (or) Demand of that row (or) column with the remaining demands (or) supplies of the undeleted columns (or) rows.
8. Go to phase-2.

Consider the transportation problem involving 3 sources & 4 destinations. The cell entries represents the cost of transportation per unit. Obtain the IBFS by using North west corner cell method.

	A	B	C	D		
Source	1	3	1	7	4	300
	2	2	6	5	9	400
	3	8	3	3	2	500
Demand	250	350	400			
		250	350	400	200	

Sol

$$\sum a_i = \sum b_j$$

$$\sum \text{Demand} = \sum \text{Supply}$$

$$1200 = 1200$$

Given transportation problem is Balanced Transportation problem.

	A	B	C	D	
1	$\frac{250}{3}$	1	7	4	50 300
2	$\frac{2}{6}$	6	5	9	400
3	$\frac{8}{3}$	3	3	2	500
	250	350	400	200	
	0				

1	50	7	4	50
2	6	5	9	400
3	3	3	2	500
	350	400	200	
	300			

	B	C	D	
2	300 6	5	9	100 400
3	3	3	2	500
	300	400	200	
	0			

	C	D	
2	100 5	9	1000
3	3	2	500
	400	200	
	300		

	C	D	
	200 3	2	500 200
3	3		
	300	200	
	0		

	D	
3	200 2	200 0
	200	

The final allocations are

250	50		
3	1	7	4
	300	100	
2	6	5	9
		300	200
8	3	3	2

The transportation cost is

$$T.C = (3 \times 250) + (1 \times 50) + (6 \times 300) + (5 \times 100) + (3 \times 300) + (2 \times 200)$$

$$T.C = 4400$$

$$\begin{aligned} \text{No of allocations} &= m+n-1 \\ &= 3+4-1 \\ &= 6 \end{aligned}$$

$$\therefore 6=6$$

Find the IBFS for the following T.P.

	D ₁	D ₂	D ₃	D ₄	Sup
S ₁	21	16	25	13	11
S ₂	17	18	14	23	13
S ₃	32	17	18	41	19
Demand	6	10	12	15	

$$\sum \text{Demand} = \sum \text{Supply}$$

$$43 = 43$$

Given transportation problem is balanced T.P.

	D ₁	D ₂	D ₃	D ₄	
S ₁	6 21	16	25	13	11 5
S ₂	17	18	14	23	13
S ₃	30	17	18	41	19
	6	10	12	15	

	D ₂	D ₃	D ₄	
S ₁	5 16	25	13	5 0
S ₂	18	14	23	13
S ₃	17	18	41	19
	10	12	15	

	D ₂	D ₃	D ₄	
S ₂	5 18	14	23	13 8
S ₃	17	18	41	19
	5	12	15	

S_2

8 14	23
S_3 18	41

15
4

S_3

D_3 4 18	D_4 41 19
4	15

S_3

D_4 15 41
15 0

The final allocations are.

6 21	5 16	25	13
17	5 18	8 14	23
32	17	4 18	15 41

total transportation cost

$$= (6 \times 21) + 5(16) + 5(18) + 8(14) + (18)4 + 4(15)$$

$$= 1095$$

$$\text{total } +.c = 1095$$

Find the TBS for the following problem by using North West Corner method

	D ₁	D ₂	D ₃	D ₄	Sup
S ₁	19	30	50	10	7
S ₂	70	30	40	60	9
S ₃	40	8	70	20	18
Dem	5	8	7	14	

$$\sum a_i = \sum b_j \Rightarrow \sum \text{Demand} = \sum \text{Supply}$$

$$34 = 34$$

5	2	50	10	7
70	6	3	60	9
40	8	4	14	18
5	8	7	14	
0	6	4	0	

transportation cost

$$= (5 \times 19) + (2 \times 36) + (6 \times 36) + (3 \times 40) + (4 \times 70) + 14 \times 20$$

$$= 1015$$

total transportation cost is 1015

$$T.C = 1015$$

Find the feasible solution for the following transportation problem

	D ₁	D ₂	D ₃	D ₄	Sup.
S ₁	6	4	1	5	14
S ₂	8	9	2	7	16
S ₃	4	3	6	2	5
Dem	6	10	15	14	

So,

$$\sum a_i = \sum b_j$$

$$\sum \text{Demand} = \sum \text{Supply}$$

$$35 \neq 35$$

Demand is not equal to supply then the problem is unbalanced transportation problem. So we need to convert unbalanced transportation problem to balanced transportation problem. So we need to add dummy row.

$$\sum a_i = \sum b_j$$

$$45 = 45$$

No, as the problem is balanced trans. problem.

	D ₁	D ₂	D ₃	D ₄	Sup.
S ₁	6	4	1	5	14
S ₂	8	9	2	7	16
S ₃	4	3	6	2	5
S ₄	0	0	0	10	10
Dem	6	10	15	14	

$$\begin{aligned}\text{Transportation Cost} &= (6 \times 6) + (8 \times 4) + (2 \times 9) + \\ &+ (14 \times 2) + (1 \times 6) + (4 \times 2) + (10 \times 10) \\ &= 128\end{aligned}$$

$$\begin{aligned}\text{allocations} &= m+n-1 \\ &= 4+4-1 \\ &= 7.\end{aligned}$$

Least Cost Cell method / Matrix minimum method.

1. Find the minimum of the values in the cost matrix.
2. Find the minimum of the supply and demand values with respect to the cell corresponding to the matrix minimum.
3. Allocate 'x' units to the cell with the matrix minimum. Also subtract 'x' units from the supply and demand values corresponding to the cell with the matrix minimum.
4. Check whether exactly one of the row (or) column corresponding to the cell with the matrix minimum has zero supply (or) zero demand respectively. If yes goto step-5 otherwise goto step-6.
5. Delete that row (or) column with respect to the cell with the matrix minimum which has the zero supply (or) zero demand goto step-7.
6. Delete both the row and column with respect to the cell with matrix minimum.
7. Check whether exactly one row (or) column is left out. If yes goto step-8. Otherwise goto step-1.

8. Match the supply (or) demand of that row (or) column with the remaining demands (or) supplies of the undeleted columns (or) rows.

9. Goto phase-II.

* Consider the following transportation problem involving three sources & 4 destinations. The cell entries represents the cost of transportation per unit. Obtain IBFS.

	D_1	D_2	D_3	D_4	Sup
S_1	3	1	7	4	300
S_2	2	6	5	9	400
S_3	8	3	3	2	500
Dem	250	350	400	200	

$$\sum a_{ij} = \sum b_j$$

$$\sum \text{Demand} = \sum \text{Supply}$$

$$1200 = 1200$$

Balanced transportation
Problem.

S₁	3	300	7	4	300
S ₂	2	6	5	9	400
S ₃	8	3	3	2	500
	250	350	400	200	
		50			

250	2	6	5	9	400	150
	8	3	3	2	500	
250		50	400	200		
0						

6	5	9	150
3	3	2	500
50	400	200	300
		0	

6	5	150
50	3	300
50		250
0	400	

5	150
250	250
400	
150	

150	150
5	

3	300 1	7	4
250 2	6	150 5	9
8	50 3	250 3	200 2

$$T.C = (1 \times 300) + (2 \times 250) + (5 \times 150) + (3 \times 50) + (250 \times 3) + (200 \times 2)$$

$$= 2850$$

Solve the IBFS least cost cell method &
North west corner cell method.

	D_1	D_2	D_3	D_4	Supply
S_1	21	16	25	13	110
S_2	17	18	14	23	13
S_3	32	17	18	41	19

Demand 6 10 12 15

Least North west Corner method
 $\sum a_i = \sum b_j$

$$\sum \text{Demand} = \sum \text{Supply}$$

$$43 = 43$$

6 21	5 16	25	13	45
17	5 18	8 14	23	138
32	17	4 18	15 41	150
6	10	12	15	
0	5	4	0	

$$T.C = (6 \times 21) + (5 \times 16) + (3 \times 15) + (6 \times 14) + (18 \times 4) + (15 \times 41)$$

$$= 1095$$

~~V. + 1095~~ 8/

Find the T.P for least cost cell method

	D ₁	D ₂	D ₃	D ₄	Supp
S ₁	6	4	1	5	14
S ₂	8	9	2	7	16
S ₃	4	3	6	2	15
Dem	6	10	15	14	

Q.2

$$\sum a_{ij} = \sum b_j$$

$$\sum \text{Demand} = \sum \text{Supply}$$

$$45 \neq 35$$

So it is unbalanced trans problem
converted in balanced transpor-
-tation problem.

for this conversion we need to
add dummy row.

Resultant transportation problem

6	4	1	5	14
8	9	2	7	16
4	3	6	2	5
6	0	0	0	10
6	10	15	14	

4	1	5	14
9	2	7	16
3	6	2	5
4	0	0	10
10	15	14	

4	5
9	7
3	2
6	0

4	1	5	14
9	2	7	16
3	6	2	5
6	15	14	

9	2	7	16
3	6	2	5
6	1	14	

9	4	15
2	5	50
6	14	

9	9	15	6
6	9		

6	9	60
6		

Final allocation

	D ₁	D ₂	D ₃	D ₄
S ₁	6	4	14	5
S ₂	8	9	10	7
S ₃	3	6	5	5
	0	40	0	0

$$\begin{array}{r} 60 \\ 60 \\ \hline 120 \\ 143 \end{array}$$

$$\text{Transportation Cost} = (6 \times 0) + (4 \times 6) + (9 \times 6) + (2 \times 1) + (5 \times 1) + (9 \times 7) + (14 \times 1)$$

$$= 0 + 0 + 54 + 2 + 10 + 63 + 14$$

$$= 143$$

Algorithms

Vogel's approximation method or VAM (or)
Penalty Method

Step ① :- Find row penalties that is the difference between the first minimum and the second minimum in each row. If the two minimum values are equal then the row penalty is zero.

Step ② :- Find the maximum amongst the column penalties that is the difference between the first minimum and the second minimum in each column. If the two minimum values are equal, then the column penalty is zero.

Step ③ :- Find the maximum amongst the row penalties and column penalties and identify whether it occurs in a row or in a column (break tie randomly). If the maximum penalty is in a row, goto step ④ otherwise goto step ⑦.

Step ④ :- ~~Allocate this~~ Identify the cell for allocation which has the least cost in that row.

Step ⑤ :- Find the minimum of the supply and demand values with respect to the selected cell.

Step ⑥ :- Allocate this minimum value to that cell and subtract this minimum from the supply and demand values with respect to the selected cell and goto step ⑩.

Step ⑦ :- Identify the cell for allocation which has the least cost in that column.

Step ⑧ :- Find the minimum of the supply and demand values with respect to the selected cell.

Step ⑨ :- Find the minimum of the supply and demand values with respect to the selected cell.

Step (9): Allocate this minimum value to the selected cell and subtract this minimum from the supply and demand values with respect to the selected cell.

Step (10): Check whether exactly one of the row or column corresponding to the selected cell has zero (supply or zero demand, respectively. If yes goto step (11) otherwise goto step (12).

Step (11): Delete the row or column which has the zero supply or zero demand and revise the corresponding row or column penalties. then goto step (13).

Step (12): Delete both the row and column with respect to the selected cell then revise the row and column penalties.

Step (13): Check whether exactly one row or column is left out. If yes goto step (14) otherwise goto step (3).

Step (14): Match the supply or the demand of the left out row or column with the remaining demands or supplies of the undeleted columns or rows.

Step (15): Goto phase II.

using Vogel's approximation method (AV) Penalty method.

3	1	7	4	300
2	6	5	9	400
8	3	3	2	500
250	350	400	200	

$$\sum a_i = \sum b_j$$

$$\sum \text{Demand} = \sum \text{Supply}$$

$$1200 = 1200$$

3	1	7	4	300	
2	6	5	9	400	150
8	3	3	2	500	
250	350	400	200		

Differences

②

③ ←

①

diff. (2 Smallest Values)

① ② ② ②

300	1	7	4	300	
6	5	9		150	
3	3	2		500	
350	400	200			

③

②

②

6	5	9	150	①
3	3	200 2	500 300	①
50	400	200		
③	②	⑦		

6	5	150	①
50 8	3	300 250	⑥
50	400		
③	②		

5	150
250 8	250
400	150

150 5	150
150	0

final allocation

	300		
3	1	7	4
250 2	6	150	9
8	3	250	200
		3	2

6	5	9	150	①
3	3	200 2	500 300	①

50 400 200
③ ② ⑦

6	5	150	①
50 3	3	300 250	⑥

50 400
③ ②

5	150
250 3	250

400
150

150 5	150
150 0	0

final allocation

	300		
3	1	7	4
250 2	6	150 5	9
8	50 3	250 3	200 2

$$P.C = 2850$$

find
matrix

, Penalty method

2	7	4	5
3	3	1	8
5	1	7	7
1	6	2	14

Supp

Demand. 7 9 18

$$\frac{16}{18} = \frac{8}{9}$$

North west method.

$$5 \times 2 = 10$$

$$3 \times 3 = 9$$

5	7	4	5
2	6	1	8
3	3	7	7
1	6	2	14

$$7 \times 0 = 0$$

$$9 \times 0 = 0$$

$$18 \times 0 = 0$$

Total allocations are $(5 \times 2) + (2 \times 3) + (3 \times 6) + (1 \times 3)$
 $(1 \times 7) + (2 \times 14)$

$$= 10 + 6 + 18 + 3 + 7 + 28 = 72$$

P.C. = 92

2	7	4	5
8	3	1	8
5	1	7	7
7	X	6	2

0 7 9 18
0

7	4	5
2	8	X
1	7	7
6	2	7

18
10

7	4	5
7	X	X
6	2	7

9 10
2

7	4	5
6	7	X

7	3	4
---	---	---

2

8

2	7
---	---

20

2
0

final allocation.

2	7	3	4
3	3	8	1
5	7	1	2
7	1	6	2

$$T.C = (1 \times 7) + (1 \times 7) + (2 \times 7) + (2 \times 7) + (1 \times 8) + (4 \times 3)$$

$$= 7 + 7 + 14 + 14 + 8 + 12$$

$$T.C = 62$$

Matrix K₁ LAM

2	7	4	5	②
3	3	1	8	②
5	7	1	10	④
1	6	2	14	⑩

7

2

9

18

①

②

①

Total allocations.

5	2	7	4	5
3	2	3	6	8
5	7	1	7	7
2	1	6	12	14
7	9	18		

Total

Transportation cost

$$= (2 \times 5) + (3 \times 2) + (1 \times 6) + (1 \times 7) + (2 \times 7) + (2 \times 12)$$

$$= 10 + 6 + 6 + 7 + 14 + 24$$

$$= 55$$

$$T.C = 55$$

Consider the transportation problem

D1	D2	D3	D4	D5	Sup	Sup
10	2	16	14	10	300	300
6	18	12	13	16	300	300
8	4	14	12	10	300	300
14	22	20	8	18	300	300

Dem 300 400 200 100 400

Find the IBFS using each of the following methods & compare their total cost

a) North west corner cell method b) Least cost cell method

c) VAM

N-W-M

$$\sum a_i = \sum b_j$$

$$\sum \text{Demand} = \sum \text{Supply}$$

$$₹ 1550 \neq ₹ 2000$$

Demand $\neq$ Supply so we need to add dummy column to convert into balanced transportation problem.

	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆ = Sup	
300 10		2	16	14	10	0	300
50 6	400 18		50 12	18	16	0	50
8	4	200 14	150 12	400 10	75 0	0	825
14	22	20	8	18	375 0	0	375
3450	400	250	150	400	450		
50	0	200	150	0	375		
0	0	0	0	0	0		

Transportation cost

$$\begin{aligned}
 &= (300 \times 10) + (50 \times 6) + (400 \times 18) + (50 \times 12) + \\
 &(200 \times 14) + (12 \times 150) + (400 \times 10) + (75 \times 0) + (375 \times 0) \\
 &= ₹ 19,700
 \end{aligned}$$

Least cost method

10	2	16	14	10	200	0	3000
350	6	18	12	16	150	0	500
8	400	25	14	12	400	0	825
14	22	20	8	18	0	0	375
350	400	250	180	400	150	0	
0	0	225	0	0	0	0	

Total transportation cost:-

$$= (350 \times 6) + (200 \times 0) + (150 \times 6) + (4 \times 400) + (14 \times 25) + (10 \times 400) + (20 \times 225) + (150 \times 8)$$

$$T.C = ₹ 13750$$

VAM

10	2	16	14	10	0	300	0
6	18	12	13	16	0	500	6
8	4	14	12	10	0	825	4
14	22	20	8	18	0	375	8
350	400	250	150	400	450		
2	2	2	1	0	0		

VAM

10	2	16	14	10	0	300	2
6	18	12	13	16	0	500	6
8	4	14	12	10	0	825	4
14	22	26	8	18	0	375	8
350	400	250	150	400	450	75	
2	2	2	4	0	0		

10	2	16	14	10	0	300	2
6	18	12	13	16	500	425	6
8	4	14	12	10	750	825	4

350	400	250	150	400	750
2	2	2	1	0	0

10	300	16	14	10	300	8
6	18	12	13	16	425	6
8	4	14	12	10	825	4

350	400	250	150	400
2	2	2	1	0
	100			

6	18	12	13	16	425	6
8	100	4	14	12	10	825

350	100	250	150	400
2	14	2	1	6

350	12	13	16	425	6
8	14	12	10	725	2

350	250	150	400
2	2	1	6

12	13	16	75	①
14	12	10	325	②
850	150	1000		
2	1	4		

12	13	75	①
14	12	325	②
250	150	175	
②	①		

75	750	175	1750
14	175	175	0
250	175		

Total allocation cost

$$= (0 \times 375) + (0 \times 75) + (2 \times 300) + (4 \times 100) + (6 \times 350) + (10 \times 400) + (12 \times 150) + (12 \times 75) + (14 \times 175)$$

Transportation cost = $\boxed{121250}$

Maximisation Case in transportation Problem.

Here the objective is to maximise the total profit for which the profit matrix is given.

First we have to convert the maximisation problem into minimisation by subtracting all the elements from the highest element in the transportation problem.

Maximisation transportation problem can be converted into minimisation problem by subtracting each transportation cost from maximum transportation cost.

For Example: The maximum transportation cost is 25, so subtract each value from 25.

There are certain types of transportation problem where the objective function is to maximise the profit instead of minimisation of cost. These problems can be solved by converting maximising into minimising.

Sunya Loshini has 3 factories X, Y & Z. It supplies goods to four dealers spread all over country. The production capacities of these factories are 200, 500 & 300 per month respectively.

	A	B	C	D	Supply
X	12	18	6	25	200
Y	8	7	10	18	500
Z	14	3	11	20	300

maximize the total net return.

Sol the given problem is maximisation transportation problem. so we need to convert into minimisation transportation problem by subtracting from the highest number.

	A	B	C	D	Supply
X	13	7	19	0	200
Y	17	18	15	7	500
Z	11	22	14	5	300
Demand	180	320	100	400	

North west Corner method.

	A	B	C	D	Supply
X	180 13	20 7	19	0	200
Y	17	300 18	100 15	100 7	500
Z	11	22	14	300 5	300
Demand	180	320	100	400	

$$\text{Transportation Cost} = (13 \times 180) + (20 \times 7) + (300 \times 18) + (15 \times 100) + (7 \times 100) + (5 \times 300)$$

$$= 2340 + 140 + 5400 + 1500 + 700 + 1500$$

	A	B	C	D	Sup
X	13	7	14	200	200 0
Y	80 17	320 18	100 15	4	500 400 3000
Z	11	22	14	5 200	300 100 0
Dem	180 80	320 0	100 0	400 200	

Transportation Cost = $(0 \times 200) + (80 \times 17) + (320 \times 18) + (15 \times 100) + (5 \times 200) + (11 \times 100)$

P.C = ₹ 10,720

VAM

	200			
13	7	14	0	200 0
17	18	15	7	500 (100 \times 1)
11	22	14	5	300

180 ~~320~~ 100 400

120

			400	
17	18	15	7	500 100
11	22	14	5	300

180 120 100 ~~400~~
0

			100
17	18	15	
180 11	22	14	300 120

18	15	100
22	109	120 20
120	100	

18	100
20	200
120	100

100	18	100
100	0	

$$\text{Transportation Cost} = (7 \times 200) + (7 \times 400) + (11 \times 180) + (14 \times 100) + (22 \times 20) + (18 \times 100)$$

$$P.C = 9820$$

Optimizing the basic feasible solution applying U-V method (or) modified method
 ↓
 modified Distribution method.

Step ①: Row 1, Row 2, Row m of the cost matrix are assigned with variables $u_1, u_2, \dots, u_m$ respectively and the column 1, column 2, ... column n are assigned with variables $v_1, v_2, \dots, v_n$ respectively.

Step ② - Check whether the number of basic cells in the set of initial basic feasible solution is equal to $m+n-1$. If yes goto step ④, otherwise goto step ③.

Step ③ - Convert the necessary number of Non-Basic cells into basic cells to satisfy the condition stated in step ②. (While doing this, sufficient care should be taken such that there is no closed loop formation with the inclusion of the new basic cells) the concept of the closed loop is explained in step ⑧.

Step ④ - Compute the values for $u_1, u_2, \dots, u_m$ and v_1, v_2 by applying the following formula to all basic cells only.

$$[u_i + v_j = c_{ij}] \quad (\text{Assume } u_1 = 0)$$

Step ⑤ - Compute penalties P_{ij} for the non-Basic cells by using the formula

$$[P_{ij} = u_i + v_j - c_{ij}]$$

Step ⑥ - Check whether all P_{ij} values are less than or equal to zero ($P_{ij} \leq 0$). If yes goto step ⑫ otherwise goto step ⑦.

Step ⑦: Identify the Non-Basic cell which has the maximum positive penalty, and term that cell as the New Basic cell.

Step ⑧:- starting from the new cell, draw a closed loop consisting of only horizontal and vertical lines passing through some basic cells (note: change of direction of the loop should be with 90° only at some basic cell).

Step ⑨:- starting from the new basic cell, alternatively assign positive (+) and Negative (-) signs at the corners of the closed loop.

Step ⑩:- find the minimum of the Allocations made amongst the Negatively signed cells.

Step ⑪:- obtain the table for the next iteration by doing following steps and then goto step ②

i) Add the minimum Allocations obtained in the previous step to all the positively signed cells and subtract minimum allocation from all the Negatively signed cells and then treat the net allocations as the allocations in the corresponding cells of the next iteration.

ii) copy the Allocations which are on the closed loop but not at the corner points of the closed loop as well as allocations which are not on the loop as such without any modification to the corresponding cells of the next iteration.

Step ⑫: The optimality is reached next the present allocations to the set of basic cells as the optimum allocations

Step ⑬:- Stop

Phase-I Finding the IBFS in the first phase.
 NWCC method
 least cost "
 VAM

Phase-II optimisation of the IBFS which is observed in phase-I.

Find the IBFS of the following transportation problem by using North west corner cell method and then optimise the solution by using U-V method.

ph.

3	1	7	4	250
2	6	5	9	250
8	3	3	2	400
200	300	350	150	

Phase-I Finding the IBFS in the first phase
 By using North west method

200 3	50 X	7	4	250 250
2	250 6	100 5	9	350 1000
8	3	250 3	150 2	400 1500
200 0	300 250 0	350 250 0	150 0	

$$Z_{AP} = \sum b_j$$

Transportation cost

$$= (200 \times 3) + (1 \times 50) + (250 \times 6) + (5 \times 100) + (150 \times 2) + (150 \times 2)$$

$$TC = 3700$$

Phase - II optimisation of the IBFS which is observed in phase - II

$u_1 = 0, u_2 = 5, u_3 = 3$ $v_1 = 3, v_2 = 1, v_3 = 0, v_4 = 1$

$u_1 = 0$	200 -3	50 1+	7	4
$u_2 = 5$	28 +	250 6-	100 5	9
$u_3 = 3$	8	3	250 3	150 2

$TC = 200$

$$P_{ij} = u_i + v_j - C_{ij} \quad (\text{Non-Basic cells})$$

$$P_{13} = 0 + 3 - 3 = 0$$

$$P_{14} = 0 + 1 - 4 = -3$$

$$P_{21} = 5 + 3 - 2 = 6 \quad \leftarrow \text{Entering Variable}$$

$$P_{24} = 5 + 1 - 9 = -3$$

$$P_{31} = 3 + 3 - 8 = -2$$

$$P_{32} = 3 + 1 - 3 = 1$$

(+ - add) - smallest Value.
(- - sub) of negative signs.

$u_1 = 0, u_2 = 5, u_3 = 3$ $v_1 = 8, v_2 = 1, v_3 = 0, v_4 = 1$

$u_1 = 0$	3	250 1	7	4
$u_2 = 5$	200 2	50 -6	100 5+	9
$u_3 = 3$	8	3	250 3	150 2

Non Basic cells

$$P_{ij} = U_i + V_j - C_{ij}$$

for all $P_{ij} \leq 0$

$$P_{11} = 0 + 3 - 3 = 0$$

$$P_{13} = 0 + 0 - 7 = -7$$

$$P_{14} = 0 - 1 - 4 = -5$$

$$P_{24} = 5 - 1 - 9 = -5$$

$$P_{31} = 3 + 3 - 8 = -2$$

$$P_{32} = 3 + 1 - 3 = 1$$

← Entering Variable.

$$U_1 = 2 \quad U_2 = 1 \quad U_3 = 11 \quad U_4 = 0$$

$$U_1 = 0$$

$$U_2 = 4$$

$$U_3 = 2$$

	3	250 1	7	4
	200 2	6	150 5	9
	8	100 3	200 3	2150

$$x = 150$$

Non Basic cells.

$$P_{11} = 0 + 2 - 3 = -1$$

$$P_{13} = 0 + 1 - 7 = -6$$

$$P_{14} = 0 + 0 - 4 = -4$$

$$P_{22} = 4 + 1 - 6 = -1$$

$$P_{24} = 4 + 0 - 4 = 0$$

$$P_{31} = 2 + 2 - 8 = -4$$

for all $P_{ij} \leq 0$ this is the

Transportation Cost

$$= (1 \times 250) + (200 \times 2) + (5 \times 150) + (50 \times 3) + (200 \times 3) + (2 \times 150)$$

$$= 2450$$

Optimal solution is 2450.

Find the IBFS of the following transportation problem by penalty method & then optimise the solution by using U-V method.

18	16	16	19	17	100
19	20	21	18	22	125
16	17	19	16	17	175
60	80	85	105	70	

Sol

$$\sum \text{Demand} = \sum \text{Supply}$$

$$400 = 400$$

$$400 = 400$$

Balanced transportation problem.

18	16	16 ⁸⁵	19	17	100	5	0
19	20	21	18	22	125		1
16	17	19	16	17	175		0
60	80	85 ⁰	105	70			
2	1	3	2	0			

18	16	19	17	15	(1)
19	20	18	22	125	(1)
16	17	16	17	75 70	(10)

60 80 105 70

(2) (1) (2) (0)

18	16	17	15	(1)
19	20	22	125	(1)
60 16	17	17	70	(1)

60 80 70

(2) (1) (0)

16	17	15	(1)
20 26	22	125	(2)
17	17	10	(0)

80 70

(1) (0)

15	17	150
22		45
17		10

70 55

22	45
10 17	100

$$\begin{array}{r} 45 \\ 22 \\ \hline 95 \\ 0 \end{array}$$

transportation cost = $(85 \times 16) + (105 \times 16) + (60 \times 16) +$
 $(20 \times 18) + (17 \times 15) + (17 \times 16) + (22 \times 45)$
 $= 7015$

Phase - I

	$V_1 = 16$	$V_2 = 15$	$V_3 = 16$	$V_4 = 16$	$V_5 = 17$	
$U_1 = 0$	18	16	16	19	17	100
$U_2 = 5$	19	20	21	18	13	125
$U_3 = 0$	16	17	19	16	17	175
	60	80	85	105	70	

Non - Basic cells.

$$P_{ij} = U_i + V_j - C_{ij}$$

$$P_{11} = 0 + 16 - 18 = -2$$

$$P_{12} = 0 + 15 - 16 = -1$$

$$P_{14} = 0 + 16 - 19 = -3$$

$$P_{21} = 5 + 16 - 19 = 2$$

$$P_{23} = 5 + 16 - 21 = 0$$

$$P_{24} = 5 + 16 - 18 = 3$$

$$P_{32} = 0 + 15 - 17 = -2$$

$$P_{33} = 0 + 16 - 19 = -3$$

$$x = 45$$

Entering Variable.

$$u_1 = 0 \quad u_2 = 2 \quad u_3 = 0$$

		$u_1 = 0$	$u_2 = 2$	$u_3 = 0$		
	18	16	85	19	17	100
	19	20	21	18	22	125
	16	17	19	16	17	175
	60	80	85	105	70	

Non-Basic cells

$$P_{11} = 0 + 16 - 18 = -2$$

$$P_{12} = 0 + 18 - 16 = 2 \quad \leftarrow \text{Entering Variable}$$

$$P_{13} = 0 + 16 - 19 = -3$$

$$P_{21} = 2 + 16 - 19 = -1$$

$$P_{23} = 2 + 16 - 21 = -3$$

$$P_{25} = 2 + 17 - 22 = -3$$

$$P_{32} = 0 + 18 - 17 = 1$$

$$P_{33} = 0 + 16 - 19 = -3$$

$$u_1 = 0 \quad u_2 = 4 \quad u_3 = 2$$

		$u_1 = 0$	$u_2 = 4$	$u_3 = 2$		
	18	15	85	19	17	
	19	65	20	21	18	22
	16	17	19	45	16	70

$$P_{ij} = u_i + v_j - C_{ij}$$

$$C_{23} = 4 + 16 - 21 = -1$$

$$C_{11} = 0 + 14 - 18 = -4$$

$$C_{25} = 4 + 15 - 22 = -3$$

$$C_{14} = 0 + 14 - 19 = -5$$

$$C_{32} = 2 + 16 - 19 = -1$$

$$C_{15} = 0 + 15 - 17 = -2$$

$$C_{21} = 4 + 16 - 21 = -1$$

Transportation cost

$$= (16 \times 15) + (185 \times 16) + (20 \times 26) + (18 \times 105) + (60 \times 16) \\ + (17 \times 45) + (17 \times 70) \\ = 6805.$$

Solve the transportation problem.

2	7	4	5
3	3	1	8
5	4	7	7
1	6	2	14
7	9	18	

$$S_{ai} = S_{bj}$$

Demand = Supply

$$34 = 34$$

North west Method.

2	7	4	5
23	3	1	8
5	34	7	7
1	6	2	14

7	9	18
7	3	14

$$\text{Transportation cost} = (2 \times 5) + (3 \times 2) + (6 \times 3) + (4 \times 3) + \\ (4 \times 7) + (2 \times 14)$$

$u_1 = 0$ $u_2 = 1$ $u_3 = 2$ $u_4 = -3$

5	2	7	4
2	3	6	3
5	3	4	4
1	6	10	2

Non Basic Cells

$$P_{ij} = u_i + v_j - c_{ij}$$

$$P_{12} = 0 + 2 - 7 = -5$$

$$P_{13} = 0 + 5 - 4 = 1$$

$$P_{23} = 1 + 5 - 1 = 5$$

Entering Variable.

$$P_{31} = 2 + 2 - 5 = -1$$

$$P_{41} = -3 + 2 - 1 = -2$$

$$P_{42} = -3 + 2 - 6 = -7$$

$u_1 = 2$ $u_2 = 2$ $u_3 = 0$

5	2	7	4
2	3	2	4
5	1	4	7
1	6	10	2

$$P_{ij} = u_i + v_j - c_{ij}$$

$$P_{12} = 0 + 2 - 7 = -5$$

$$P_{13} = 0 + 0 - 4 = -4$$

$$P_{31} = 2 + 2 - 5 = -1$$

$$P_{41} = 2 + 2 - 1 = 3$$

$$P_{42} = 2 + 2 - 6 = -2$$

$u_1 = 0$
 $u_2 = -2$
 $u_3 = -1$
 $u_4 = -1$

5	2	7	4
3	2	3	1
5	7	4	7
2	1	6	12

Non Basic cells

$$P_{12} = 0 + 5 - 7 = -2$$

$$P_{13} = 0 + 3 - 4 = -1$$

$$P_{21} = -2 + 2 - 3 = -3$$

$$P_{31} = -1 + 2 - 5 = -4$$

$$P_{33} = -1 + 3 - 7 = -5$$

$$P_{42} = -1 + 5 - 6 = -2$$

for all $C_{ij} \leq 0$

$$\begin{aligned}
 \text{Transportation cost} &= (5 \times 2) + (3 \times 2) + (1 \times 6) + (4 \times 7) \\
 &\quad + (2 \times 12) + (1 \times 2) \\
 &= 10 + 6 + 6 + 28 + 24 + 2 \\
 &= 76
 \end{aligned}$$

Transportation cost = 76

Transportation Degeneracy: UV method.

In a transportation problem, no of allocation is less than $m+n-1$ that is called transportation Degeneracy

$$m+n-1 < \text{No of allocations}$$

Degeneracy occurs in two cases

- Case:1 Degeneracy at IBFS
Case:2 Degeneracy at subsequent Iteration.
(In phase-II In UV method)

Case:1 Obtain the IBFS using the Least Cost cell method & then optimise the solution by using UV method

10	2	3	15	9	35	Sup
5	10	15	2	4	40	
15	5	14	7	15	20	
				8	30	

$$(5 \times 20) + (2 \times 20) + (3 \times 25) + (15 \times 13) + (9 \times 25) + (2 \times 8) + (4 \times 30) + (40 \times 10) + (20 \times 15) = 1250$$

Phase-I Finding the IBFS
 $\sum C_i = \sum b_j$
 $125 = 125$

By Using Least Cost

10	20	15	15	9	35	180
5	10	15	10	2	4	30
20	15	14	7	15	20	0
20	15	25	13	25	58	30
20	0	10	10	35	70	0

Transportation

$$m+n-1 = \text{No of allocations}$$

$$4+5-1 = 7$$

$$8 \neq 7 \quad (\text{Transp. discrepancy})$$

$$\text{Transportation Cost} = (2 \times 20) + (3 \times 15) + (2 \times 10) + (4 \times 30) + (20 \times 15) + (13 \times 25) + (5 \times 8)$$

$$\text{Transportation Cost} = 890$$

Phase 2

$$U_1 = 12 \quad U_2 = 2 \quad U_3 = 3 \quad U_4 = -4 \quad U_5 = -2$$

$U_1 = 0$	10	20	2	3	15	15	9
$U_2 = 6$	5	10	15	10	2	30	4
$U_3 = 3$	15	5	14	7	15		
$U_4 = 10$	20	15	13	25	5	8	

$$c_{11} = 2$$

$$c_{22} = -2$$

$$c_{35} = -14$$

$$c_{14} = -19$$

$$c_{23} = -6$$

$$c_{41} = 2$$

$$c_{15} = -11$$

$$c_{33} = -6$$

$$c_{42} = -3$$

$$c_{21} = 13$$

$$c_{34} = -8$$

$$c_{44} = -19$$

$$u_1 = -1 \quad u_2 = 2 \quad u_3 = 3 \quad u_4 = -4 \quad u_5 = -2$$

$u_1 = 0$	10	2	(E 35) 3	15	9
$u_2 = 6$	5	10	15	10/2	10/4
$u_3 = 3$	15	20/5	14	7	15
$u_4 = 10$	20	15	5/13	25	25/8

$$7 \neq 8$$

$$P_{ij} = u_i + v_j + c_{ij}$$

$$c_{11} = -11$$

$$c_{31} = -13$$

$$c_{42} = -3$$

$$c_{14} = -19$$

$$c_{33} = -8$$

$$c_{44} = -19$$

$$c_{15} = -11$$

$$c_{34} = -8$$

$$c_{22} = -2$$

$$c_{35} = -14$$

$$c_{23} = -6$$

$$c_{41} = -11$$

All $P_{ij} \leq 0$, so it is an optimization solution

$$Z = (35 \times 3) + (20 \times 5) + (10 \times 2) + (10 \times 4) + (6 \times 5) + 10$$

$$= ₹ 630$$

Handwritten mathematical work on a page, featuring several rows of numbers and calculations. A large diagonal watermark "GATE COLLEGE" is visible across the page.

Top Section:

	19	30	50	10	Σ	LFAM
	70	30	40	60	9	779
	40	8	70	20	18	743
Σ	5	8	783	14		

Middle Section:

	6	3	5	4	Σ	(LCCM)
	5	9	2	7	15	150
	5	7	8	6	8	149
Σ	7	12	17	9		

Bottom Section:

	5	8	6	6	3	Σ	LFAM
	4	7	7	6	5	5	97
						9	92
	8	4	6	6	4		
Σ	4	4	5	4	8		

Transportation Degeneracy Case: 2

Transportation Degeneracy at subsequent Iterations

	8	5	6	120
	15	10	12	180
	3	9	10	180
D	180	80	50	

120	8	5	6	120
30	15	50	10	12
3	30	9	50	10

180 80 50

30 30 0

$$\begin{aligned}
 & (120 \times 8) + (30 \times 15) + (50 \times 10) + (30 \times 9) + (50 \times 10) \\
 &= 960 + 450 + 500 + 270 + 500 \\
 &= 2680
 \end{aligned}$$

Phase: 2

Phase: 2

$u_1 = 0$ $v_1 = 8$ $v_2 = 3$ $v_3 = 4$

	120			
	8	5	6	
$u_2 = 7$	30	50 + 10	12	
	15			
$u_3 = 6$	3	30	9	50 10

P. KEERTHI, MBA, M.S.C (MATHEMATICS), ASST. PROF, GA

Non Basic Cells $P_{ij} = u_i + v_j - C_{ij}$

$$P_{12} = 0 + 3 - 5 = -2$$

$$P_{13} = 0 + 4 - 6 = -2$$

$$P_{23} = 7 + 4 - 12 = -1$$

$$P_{31} = 6 + 8 - 3 = 11 \leftarrow \text{Entering Variable}$$

$$u_1 = 8 \quad u_2 = 5 \quad u_3 = 15$$

$$u_1 = 0$$

$u_1 = 0$	10 8	8 5	6 +
$u_2 = 5$	15	80 10	12
$u_3 = -5$	30 3	9	50 10

$$P_{13} = 0 + 15 - 6 = 9$$

Entering Variable

$$P_{21} = 5 + 8 - 15 = -2$$

$$P_{23} = 5 + 15 - 12 = 8$$

$$P_{32} = -5 + 5 - 9 = -9$$

$$u_1 = 8 \quad u_2 = 5 \quad u_3 = 6$$

$$u_1 = 0$$

$u_1 = 0$	70 8	8 5	6 50
$u_2 = 5$	15	80 10	12
$u_3 = -5$	80 3	9	10

$$P_{21} = 5 + 8 - 15 = -2$$

$$P_{23} = 5 + 6 - 12 = -1$$

$$P_{32} = -5 + 5 - 9 = -9$$

$$P_{33} = -5 + 6 - 10 = -9$$

for all $C_{ij} \leq 0$

$$T.C = (70 \times 8) + (80 \times 10) + (50 \times 6) + (80 \times 3)$$

$$= 560 + 800 + 300 + 240$$

19	30	50	10
70	30	40	60
40	8	70	20

5 8 7 14

$$\sum a_i = \sum b_j$$

$$34 = 34$$

15	19	30	50	10
70	30	40	60	
40	8	70	20	

70 8 7 14
(21) (0) (0) (10)

30	50	2	10
30	40	60	
8	70	20	

8 7 14

30	40	60
8	70	¹² 20

9 (10)

~~18~~ (12)

8

7

~~12~~ 0

(22)

(30)

(40)

30	40
6 8	70

9 10

~~60~~ 628₂

7

22

30

2	30	40
---	---------------	----

9 7

2

7

7	40
---	----

~~70~~~~70~~

Transportation Cost

$$= (2 \times 10) + (5 \times 19) + (20 \times 12) + (6 \times 8) + (2 \times 36) + (40 \times 7)$$

$$= 743$$

19	30	50	10	7	(9)
70	80	40	60	9	(10)
40	8	70	20	18 10	(12)

5 80 7 14
(21) (22) (10) (10)

5 19	50	10	7	2	(9)
70	40	60	9		(20)
40	70	20	10		(20)

70 7 14
(21) (10) (10)

50	10	2	40
40	60	9	20
70	20	10	50

7 14
(10) (10)

50	10	26	40
40	60	9	20

$$\begin{aligned}
 \text{P.C.} &= (8 \times 8) + (5 \times 19) + (20 \times 10) + (10 \times 2) + \\
 &\quad (7 \times 40) + (60 \times 2) \\
 &= 779.
 \end{aligned}$$

Phase: 2

$u_1 = 0$ $u_2 = 50$ $u_3 = 10$

$v_1 = 19$ $v_2 = 2$ $v_3 = 10$ $v_4 = 10$

5	19	30	50	10	7
		30	40	60	9
		8	70	20	18
5	8	7	14		

Non-Basic cells.

$$P_2 = 0 + 2 - 32 = -34$$

$$P_3 = 0 - 10 - 50 = -60$$

$$P_5 = 0 + 10 - 7 = 3$$

$$P_{21} = 50 + 19 - 70 = -1$$

$$P_{22} = 50 - 2 - 30 = 18 \leftarrow \text{Entering variable.}$$

$$P_{25} = 50 + 10 - 9 = 51$$

$$P_{31} = 10 + 19 - 40 = -11$$

$$P_{33} = 10 - 10 - 70 = -77$$

$u_1 = 19$ $u_2 = 40$ $u_3 = 50$ $u_4 = 12$

$u_1 = 0$	5	19	30	2	50	10
$u_2 = -10$		70	+2 30	4	40	60
$u_3 = 32$		40	6	8	70	20

Non-Basic cells.

$$P_{12} = 0 + 40 - 30 = 10$$

$$P_{14} = 0 + 12 - 10 = 2$$

$$P_{21} = -10 + 19 - 70 = -69$$

$$P_{24} = -10 + 12 - 60 = -58$$

$$P_{31} = 32 + 19 - 40 = 11$$

$$P_{33} = 32 + 50 - 70 = 12 \quad \leftarrow \text{Entering Variable}$$

$u_1 = 19$ $u_2 = 40$ $u_3 = 50$ $u_4 = 0$

$u_1 = 0$	5	19	30	2	50	10	
$u_2 = -10$		70	8	30	1	40	60
$u_3 = 20$		40	6	8	70	12	20

Non Basic cells.

$$P_{12} = 0 + 40 - 30 = 10$$

$$P_{14} = 0 + 0 - 10 = -10$$

$$P_{21} = -10 + 19 - 70 = -61$$

$$P_{24} = -10 + 0 - 60 = -70$$

$$P_{31} = 20 + 19 - 40 = -1$$

$$P_{32} = 20 + 40 - 8 = 52$$

Problems

	S_1	S_2	S_3	S_4	D_1	D_2	
S_1	0	4	20	5	25	12	100 + 800
S_2	10	0	6	10	5	20	200 + 800
S_3	15	20	0	8	45	7	150 + 800
S_4	20	25	10	0	30	6	250 + 800
D_1	20	18	60	15	0	10	800
D_2	10	25	30	23	4	0	800
	800	800	800	800	800 + 800	450 + 800	800

$$S_{a_i} = S_{b_j}$$

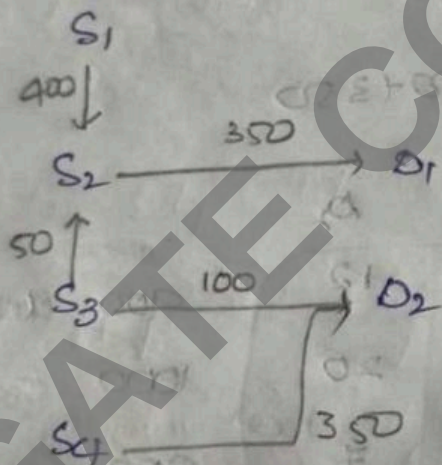
Demand = Supply

$$b = 100 + 200 + 150 + 350$$

	S_1	S_2	S_3	S_4	D_1	D_2	
S_1	0	4	20	5	25	12	100 + 800
S_2	10	0	6	10	5	20	200 + 800
S_3	15	20	0	8	45	7	150 + 800
S_4	20	25	10	0	30	6	250 + 800
D_1	20	18	60	15	0	10	800
D_2	10	25	30	23	4	0	800
	800	800	800	800	1150	1250	400 + 1000

final allocations

	S_1	S_2	S_3	S_4	D_1	D_2
S_1	400	100				
S_2	0	4			350	
S_3		650	5			
S_4		0	8			
D_1		50	20	800		100
D_2		20	0	0	350	
				800		6
				0	800	
					0	800
						0



$$\begin{aligned}
 \text{Transportation cost} &= (4 \times 100) + (350 \times 5) + (20 \times 50) \\
 &\quad + (7 \times 100) + (6 \times 350) \\
 &= 5950
 \end{aligned}$$

Assignment programming Problem (APP)

Introduction:

Assignment problem is used when a particular job (or) machine is to be assigned to a particular worker based on his training and professional level (or) a project manager has to deposited to a particular project based on his qualification, experience and spl exposure to that spl type of project.

Meaning:

An assignment problem is to assign in number of origins (jobs) to the equal 'n' number of destinations at a minimum cost (or) maximum profit.

Thus the problem is how the assignments should be made so as to optimize the given objective.

Applications:

Some of the problems is how the assignment technique may be useful are.

- * Assignment of workers to machines
- * Assignment of salesman to different sales Areas
- * Assignment of clerks to various checkout counters
- * Assignment of classes to rooms
- * Assignment of vehicles to routes
- * Assignment of contracts to bidders
- * Assignment of faculty to subjects

Mathematical model of App:-

Let there be 'n' number of sources of facilities supply or workers $w_1, w_2, \dots, w_n$ having a_i ($i=1, 2, \dots, n$) units of supply (capacity) respectively to be assigned among 'n' number of activities or jobs $J_1, J_2, \dots, J_n$ with b_j ($j=1, 2, \dots, n$) units of demand or requirement respectively.

Let c_{ij} be the effectiveness (cost, profit, time etc) of assigning each facility to each job to determine the assignment schedule so to optimize (maximize profit, minimize cost / time). the given objective. Mathematically the Assignment problem may be stated as follows

Objective function :-

$$\text{Maximize or minimize } Z = \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$$

S.T.C subject to constraints.

$$\text{Supply constraint } \sum_{j=1}^n x_{ij} = a_i; i=1, 2, \dots, n$$

$$\text{Demand constraint } \sum_{i=1}^n x_{ij} = b_j; j=1, 2, 3, \dots, n$$

Restrictions :-

$$x_{ij} \geq 0 \text{ (Non-Negative variables)}$$

Generalised format of Assignment problems :-
operators.

	1	2	...	j	...	m
1	t_{11}	t_{12}	...	t_{1j}	...	t_{1m}
2	t_{21}	t_{22}	...	t_{2j}	...	t_{2m}
...						
b						

t_{11}	t_{12}	t_{1j}	t_{1n}
t_{m1}	t_{m2}	t_{mj}	t_{mn}

where m = number of jobs as well as operators

t_{ij} = processing time of the job i , if it is assigned to the operator j .

"The main object is to assign the jobs to the operators such that the total processing time is minimised"

Types of Assignment problems:-

1. Balanced Assignment problem
2. unbalanced Assignment problem

1. Balanced Assignment problem:- if the number of rows (jobs) is equal to the number of columns (operators) then the problem is termed as a balanced assignment problem.

2. unbalanced Assignment problem:- if the number of rows is not equal to the number of columns (operators) then the problem is termed as a unbalanced assignment problem.

Hungarian method:-

An assignment problem can be easily solved by applying Hungarian method which consists of two phases. In the first phase, row reductions and column reductions are carried out. In the second phase, the solution is optimised on iterative basis.

Algorithm for Hungarian Method:-

Phase - I Row & column Reductions.

Step ①:- Consider the given cost matrix

Obtain the next matrix by subtracting the minimum value of each row from the entries of that row.

Step ③: Obtain the next matrix from the entries of minimum value of each column from the entries of that column. Now treat this matrix as the input for phase-II

PHASE-II — optimisation of the problem

Step ③: Draw a minimum number of lines to cover all the zeros of the matrix. the procedure for drawing minimum number of lines involves the following steps.

(i) Row Scanning :-

a) starting from the first row is there exactly one zero in that row mark a square around that zero entry and draw a vertical line passing through that zero otherwise skip that row.

b) After scanning the last row check whether all the zeros are covered with lines. If yes goto step ④ otherwise do column scanning.

(ii) Column Scanning :-

a) starting from the first column is there exactly one zero in that column mark a square around that zero entry and draw a horizontal line passing through the zero otherwise skip that column.

b) After scanning the last column check whether all the zeroes are covered with lines. If yes goto step ④ otherwise do row scanning.

Step ④: Check whether the number of squares marked is equal to the number of rows of the matrix if yes goto step ⑦ otherwise goto step ⑤.

Step ⑤: Identify the minimum value of the undeleted cell values obtain the next matrix by following the steps mentioned below.

(i) copy the entries on the lines but not on the intersection points of the present matrix as such without any modification to the corresponding positions of the next matrix.

(ii) copy the entries at the intersection points of the present matrix after adding the minimum undeleted cell value to the corresponding positions of the next matrix.

(iii) Subtract the minimum undeleted cell value from ~~all~~ the undeleted cell values and then copy them to the corresponding positions of the next matrix.

Step ⑥: Go to Step ③.

Step ⑦: Treat the solution as marked by the squares as the optimal solution.

Hungarian Method

Types of assignment problems:

1) Balanced assignment problem

2) Unbalanced assignment problem

Hungarian Method

Phase: 1 Row & column reductions

Phase: 2 Optimisation of the problem.

Solve the following assignment problem using Hungarian method: The matrix entries represent

processing time in hrs operator

	1	2	3	4	5
Job 1	9	11	14	11	7
2	6	15	13	13	10
3	12	13	6	8	8
4	11	9	10	12	9
5	7	12	14	10	14

optimality condition is

No. of rows is equal to No. of squares.

This is balanced assignment problem bcoz no. of row is equal to no. of columns.

Sol

Step: 1 Identification of min row number.

9	11	14	11	7	7
6	15	13	13	10	6
12	13	6	8	8	6
11	9	10	12	9	9
7	12	14	10	14	7

Row Reduction Matrix

2	4	7	4	0
0	9	7	7	4
6	7	0	2	2
2	0	1	3	0
0	5	7	3	7

Identify the least element in column the resultant matrix is column reduction matrix after reducing the least element from the respected column.

Column reduction matrix we should do

2	4	7	2	0
0	9	7	5	4
6	7	0	0	2
2	0	1	1	0
0	5	7	1	7

row scanning
(or) column scanning

Optimality Condition is not satisfied because No. of row is not equal to no of squares then we go to Iterations

To go further we need to select the minimum number from the undeleted elements

* The minimum element should be added to the intersected elements, subtracted from the undeleted elements & copied from the deleted elements.

The resultant Iteration is.

	1	2	3	4	5
1	2	4	6	1	10
2	10	9	6	4	7
3	7	8	10	0	3
4	2	10	0	0	0
5	0	5	6	0	7

Optimality condition is satisfied because the no. of squares is equal to no. of rows

$$\text{i.e., } 5 = 5$$

Job & operators allocation

operator tym.

Job

1

2

3

4

5

operator

5

6

6

2

4

7

6

6

9

10

38

Job
matrix
has.

the following assignment matrix. The entries represents the processing time in

	1	2	operator 3	4	5
1	10	12	15	12	8
2	7	16	14	14	11
3	13	14	7	9	9
4	12	10	11	13	10
5	8	13	15	11	15

	1	2	operator 3	4	5	Row minima
1	10	12	15	12	8	8
2	7	16	14	14	11	7
3	13	14	7	9	9	7
4	12	10	11	13	10	10
5	8	13	15	11	15	8

Row Reduction matrix.

2	4	7	4	0
0	9	7	7	4
6	7	0	2	2
2	0	1	3	0
0	5	7	3	7

Column

Reduction

Matrix

2	4	7	2	10
10	9	7	5	4
6	7	10	0	2
2	10	1	1	0
0	5	7	1	7

No. of square is not equal to no. of rows

$$4 \neq 5$$

Then we go to next iteration

$$x=1$$

$$\text{Intersect} = +$$

$$\text{deleted} = \text{Copy}$$

$$\text{undelated} = c -$$

	1	2	3	4	5
1	2	4	6	10	
2	10	9	6	4	4
3	7	8	10	0	3
4	2	10	0	0	0
5	0	5	6	10	7

Optimality is reached here no. of rows = no. of squares

Job	operator	lym.
1	5	8
2	1	7
3	3	7
4	2	10
5	4	7

Unbalanced assignment problem

The no. of row is not equal to no. of columns then assignment problem is called as the unbalanced assignment problem. So we convert unbalanced assignment problem by adding dummy row or dummy column.

	Sub	1	2	3	4	5
Faculty	1	30	39	31	38	40
	2	43	37	32	35	38
	3	34	41	33	41	34
	4	39	36	43	32	36
	5	32	49	35	40	37
	6	36	42	35	40	42

Given that the no. of rows is not equal to no. of columns i.e., $6 \neq 5$

So we add dummy column to balance the problem.

	30	39	31	38	40	0	Row minima
						0	0
	43	37	32	35	38	0	0
	34	41	33	41	34	0	0
	39	36	43	32	36	0	0
	32	49	35	40	37	0	0
	36	42	35	40	42	0	0

Row Reduction Matrix

30	39	31	38	40	0
43	37	32	35	38	0
34	41	33	41	34	0
39	36	43	32	36	0
32	49	35	40	37	0
36	42	35	40	42	0

Column
Red.

30	36	31	32	34	0
0	3	0	6	6	0
13	1	1	3	4	0
4	5	2	11	0	0
9	0	13	0	2	0
2	13	5	8	3	0
6	6	5	8	8	0

$$x = 1$$

12	0	0	2	4	0
3	4	1	10	0	0
9	0	13	0	3	0
1	12	4	7	3	0
5	5	4	7	8	0

No of Row $\neq$ No of Squares

$$x = 1$$

0	3	0	6	7	2
12	6	0	2	7	1
3	4	1	10	0	1
9	0	13	0	3	1
0	11	3	6	2	0
4	4	3	6	7	0

Faculty

Sub

tyr

1

3

31

2

2

37

3

5

34

4

4

32

5

1

32

6

6

10

Application of diagonal selection
 Solve the following Assignment problem using
 Hungarian method. The matrix entries represent
 the processing time in hrs

Operator

	A	B	C	D	Row minimum
1	5	3	2	8	2
2	7	9	2	6	2
3	6	4	5	7	4
4	5	7	7	8	5

Row Reduction Matrix

3	1	0	5
5	7	0	4
2	0	1	3
0	2	2	3
0	0	0	3

Column Reduction Matrix

3	7	0	2
5	7	0	1
2	0	1	0
0	2	2	0

$$3 \neq 4$$

So optimisation is not reached so go to Iteration -1.

$n=1$

	A	B	C	D
1	2	0	0	2
2	4	6	0	0
3	2	0	2	0
4	0	2	3	0

So there is no possibility to select '0' in Row scanning & Column scanning. then

Select diagonal zero's

No of Rows & No of Columns

optimality is Reached

Job	operator	time
1	2	3
2	3	2
3	4	1
4	1	5

17 hrs.

Assignment problem

Convert it into a minimisation problem by subtracting all the elements from the largest element.

Solve the following assignment problem by using Hungarian method to maximize sales

	Territories			
	I	II	III	IV
Salesman A	45	38	30	22
B	35	29	20	14
C	35	29	20	14
D	27	20	15	10

Sol the given assignment problem is maximisation problem type so we convert maximisation problem into minimisation problem by subtracting maximum value from the all the elements i.e., 45

	Resultant problem is			
	I	II	III	IV
A	0	7	15	23
B	10	16	25	31
C	10	16	25	31
D	18	25	30	35

Row

0	7	15	23
0	6	15	21
0	6	15	21
0	7	12	17

0 6 12 17

Column Reduction Matrix.

0		3	6
0	0	3	4
0	0	3	4
0		0	0

No of square $\neq$ No of rows go to

Iteration $n=3$ Resultant Matrix.

	I	II	III	IV
A	0	1	0	3
B	0	0	0	1
C	0	0	0	1
D	3	4	3	0

Job	operator	time
Scull man	ten	45
A	I	29
B	II	20
C	III	10
D	IV	104

No of rows = No of columns.

Processing time = 104 hours

A department of a company has 6 employees with 5 jobs to be performed. The time in hrs that each man takes to perform each job is given in the effectiveness matrix.

	1	2	3	4	5
A	10	5	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

How should the jobs to be allocated 1 per employee so as to minimize the total man hours.

	1	2	3	4	5	
A	10	5	13	15	16	5
B	3	9	18	13	6	3
C	10	7	2	2	2	2
D	7	11	9	7	12	7
E	7	9	10	4	12	4

5	0	8	10	11
0	6	15	10	3
8	5	0	0	0
0	4	2	0	5
3	5	6	0	8

0 0 0 0 0

column Reduction Matrix

5	0	8	10	11
0	6	15	10	3
8	5	0	0	0
0	4	2	0	5
3	5	6	0	8

$\lambda = 3$

5	0	6	10	9
0	6	13	10	1
10	7	10	2	0
0	4	0	0	3
3	5	4	0	6

Job	Emp	Time
A	2	5
B	1	3
C	5	6
D	3	2
E	4	7

23 hrs

5 men are available to do 5 different jobs from past records the time in hrs that each man takes to do each job is known & given in the following table.

	Job					
	1	2	3	4	5	
A	2	9	2	7	1	(1)
B	6	8	7	6	1	(1)
C	4	6	5	3	1	(1)
D	4	2	7	3	1	(1)
E	5	3	9	5	1	(1)

find that how men should be assigned to jobs in way that will minimize the total time taken.

Row Reduction matrix

1	8	1	6	0	
5	7	6	5	0	
3	5	4	2	0	
3	1	6	2	0	
4	2	8	4	0	
(1)	(1)	(1)	(2)	(0)	

Column Reduction Magic

0	7	0	4	0
4	6	5	3	0
2	4	8	0	0
2	0	5	0	0
3	1	7	2	0

$$\lambda = 2$$

0	9	0	6	2
2	6	3	3	0
0	4	1	0	0
0	0	3	0	0
1	1	5	2	0

$$\lambda = 1$$

0	9	0	6	3
1	5	2	2	0
0	4	1	0	1
0	0	3	0	1
0	0	4	1	0

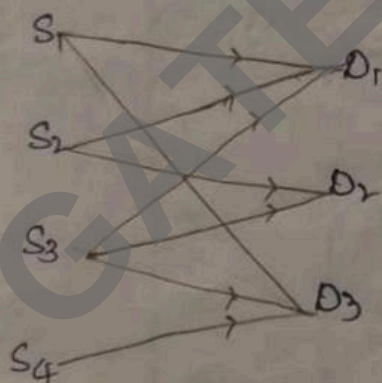
Trans-shipment

Trans-shipment problem.

In a transshipment problem shipment of commodity takes place among sources and destinations. But instead of direct shipment to destinations the commodity can be transported to a particular destination through one or more intermediate (or) transshipment points. Each of these points in turn supply to other points. Thus when the shipment goes from destination to destination and from source to source we have a transshipment problem.

Example: four sources & 3 destination problems

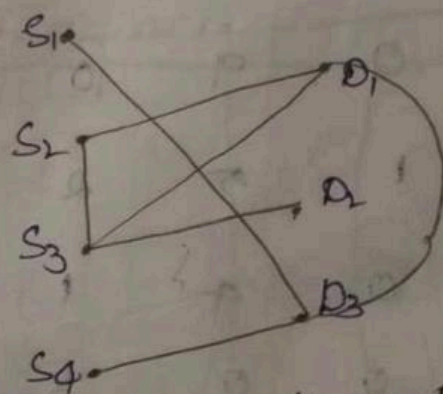
Source Destination



a) transportation

Source

Destination



b) trans-shipment

Since flow of commodity can be in both directions arrows are not shown in fig (b). The solution to this problem can be obtained by using transportation problem.

A trans-shipment problem allows shipment between point and demand. It may also contain trans-shipment points through which goods may be shipped on their way from a supply point to demand point. Using the following method a trans-shipment problem may be transposed into a transportation problem.

Process / Algorithm for trans-shipment problem

Step:1 If necessary add a dummy demand point to a balanced problem shipment to a dummy from a point to itself will of course have a zero shipping cost

Let $S =$ total availability supply.

Step:2 Construct transportation table creating route for each supply point. Each transportation problem has a row for each supply point and a column for each demand point and a trans-shipment point. Each supply point we have a supply equal to each original supply & each demand point will have a demand to its original demand

Step:3 At $S =$ total availability supply, then each trans-shipment point will have a supply equal to original supply $f(s)$ & demand equal to $f(s)$

Difference between transportation & Trans-shipment

Transportation

Trans-shipment

* Transportation problem is a model in a spl case of LPP. that details with commodity from source through transit nodes to destination.

* Transportation allows to make decisions regarding logistic of competition with immediate feedback

* Transshipment model can be converted into transport problem using ideal buffer.

* Transportation problem has both feasible solution & optimal solution.

* It has supply on column values and demand values appears on row values

* Both demand & supply appears both row and column.

dates man ship method. (Travelling Sales man) Problem

- * There are no. of cities a sales man must visit ex: A, B, C, D, etc...
- * The distance or type of cost between every pair of cities is given
- * The sales man ~~starts~~ starts from his home city, he must visit every city exactly once and returns to his home city

* The problem is to find the most shortest distance (or type) (or cost).

Solution for Salesmanship (or) Travelling sales man Problem

Phase: I Travelling sales man problem can be first solved as assignment problem by using Hungarian method to find optimum solution

* then TSP Condition.

* If the condition is satisfied, then assignment problem solution will be the optimum solution even for TSP problem.

* If not go to phase - II

Phase: II

Method: I the solution can be adjusted by inspection

* II - from a single circuit

III - The Iterative procedure - Branch & bound method

A travelling sales man has planned to visit 4 cities. He would like to start from a particular city visit each city only once & return to the starting city. The travelling cost in Rs is given in the table below. Find the least cost route.

	TO	A	B	C	D
from A	0	25	75	45	
B	35	0	150	25	
C	35	40	0	15	
D	65	75	130	0	

Sol

	A	B	C	D	Row min
A	0	25	75	45	25
B	35	0	150	25	25
C	35	40	0	15	15
D	65	75	130	0	65

x	0	50	20
10	x	125	0
20	25	x	0
0	10	65	x
0	0	50	0

10	10	0	20
10	x	75	0
20	25	x	0
0	10	15	x

$$[x=10]$$

10	0	0	30
10	x	65	0
20	15	x	0
10	0	5	x

$$x=10$$

	A	B	C	D
A	0	0	0	40
B	0	0	55	0
C	10	5	0	0
D	0	0	5	0

Route

A $\rightarrow$ CC $\rightarrow$ DD $\rightarrow$ BB $\rightarrow$ A

$$A \rightarrow C \rightarrow D \rightarrow B \rightarrow A \Rightarrow 75 + 15 + 75 + 35 = 200$$

The travelling sales visits only one firm
optimum solution obtained

Condition

The sales man starts from his home city, he must visit every city exactly once & return to his home city.

A salesman wants to visit city A, B, C, D, E. He doesn't want to visit any city twice bfr completing his tour of all the cities & wishes to return the point of starting journey. Cost of going from one city to another is given in the table below. Find the least cost route.

	TO					
	A	B	C	D	E	
A	∞	2	5	8	1	1
B	6	∞	3	9	2	3
C	8	7	∞	4	8	4
D	13	14	7	∞	5	4
E	1	3	2	8	∞	1

Row Reduction Matrix

	A	B	C	D	E
A	∞	1	4	7	0
B	4	∞	1	7	0
C	4	3	∞	0	4
D	9	0	3	∞	1
E	0	2	1	7	∞

column Reduction Matrix

	A	B	C	D	E
A	∞	1	3	7	$\boxed{0}$
B	4	∞	$\boxed{0}$	7	0
C	4	3	∞	$\boxed{0}$	4
D	9	$\boxed{0}$	2	∞	1
E	$\boxed{0}$	2	0	7	∞

The travelling sales man condition is not satisfied
 the sales person visit same city twice so go to phase-II for optimal solution
 Assignment solution

∞	1	3	7	$\boxed{0}$
4	∞	$\boxed{0}$	7	0
4	3	∞	$\boxed{0}$	4
9	$\boxed{0}$	2	∞	1
$\boxed{0}$	2	0	7	∞

$\Rightarrow$ optimum solution is not satisfied then go to

Iterations

1st Method $\Rightarrow$ Inspection method $\Rightarrow$ select next least value i.e., '1'

	A	B	C	D	E
A	∞	$\boxed{1}$	3	7	0
B	4	∞	$\boxed{0}$	7	0
C	4	3	∞	$\boxed{0}$	4
D	9	0	2	∞	$\boxed{1}$
E	$\boxed{0}$	2	0	7	∞

A-B	2
B-C	3
C-D	4
D-E	5
D-A	1
	15

Single Circuit Method:

A salesman wants to visit cities A B C D E. He doesn't want to visit any city twice for completing his tour of all the cities & wishes to return to the point of start a journey. Cost of going from one city to another is given in the table below. find the least cost.

	A	B	C	D	E	
A	∞	2	5	8	1	1
B	6	∞	3	9	2	2
C	8	7	∞	4	8	4
D	13	4	7	∞	5	4
E	1	3	2	8	∞	1

Assignment

Row reduction matrix

	A	B	C	D	E
A	∞	1	4	7	0
B	4	∞	1	7	0
C	4	3	∞	0	4
D	9	0	3	∞	1
E	0	2	1	7	∞
	0	0	1	0	0

Column Reduction Matrix.

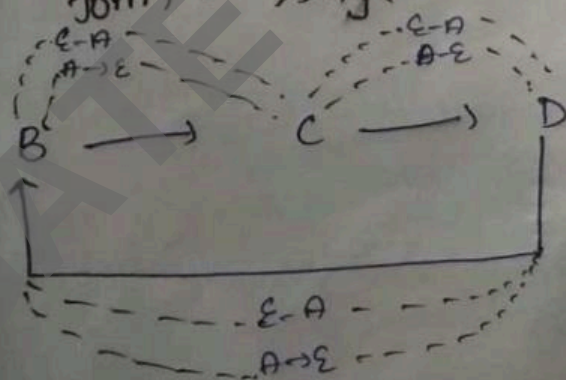
	A	B	C	D	E
A	∞	∞	9	7	∞
B	4	∞	∞	∞	0
C	4	3	∞	∞	7
D	9	∞	2	∞	1
E	∞	2	0	7	∞

$A \rightarrow E$: the travelling sales man
 $E \rightarrow A$: condition is not satisfied.
 $B \rightarrow C$: the sales person visited same
 $C \rightarrow D$: city twice so go to phase-II
 $D \rightarrow B$: 4 for optimal solution.

$$A \rightarrow E \rightarrow D$$

$$B \rightarrow C \rightarrow D \rightarrow B$$

Phase-II form a single circuit!



$$\begin{aligned}
 BA \text{ \& \& } EC &= 4 + 0 = 4 \\
 BE \text{ \& \& } AC &= 0 + 3 = 3 \\
 CA \text{ \& \& } ED &= 4 + 7 = 11 \\
 CE \text{ \& \& } AD &= 4 + 7 = 11 \\
 DA \text{ \& \& } EB &= 9 + 2 = 11 \\
 DE \text{ \& \& } AB &= 1 + 1 = 2.
 \end{aligned}$$

$$A \rightarrow B \rightarrow C \rightarrow D \rightarrow E$$

$$1 + 1 + 3 + 4 + 4 = 13 + 2 = 15$$

Introduction:-

- Basically it is an optimisation problem
- * It divides a given optimisation problem into a number of sub problems known as "stages".
 - * At each stage there is no. of alternatives known as "states".
 - The no. of states may vary from stage to stage.
 - * Each stage is an optimisation problem. Therefore we have to solve them at each stage.
 - * The optimisation problem in each stage is dependent of sub problem in the previous stage.
 - * The optimum solution of 'n' stages gives the final solution of original problem.

Definition:-

* (Dynamic programming problem is optimisation problem, which divides the original problem into number of sub problems, by integrating the solution of sub problem gives the optimum solution of the original problem.)

NO (By using BELLMAN'S principle of optimality to solve the given dynamic programming problem.) NO

Dynamic programming is a special kind of optimisation technique which subdivides an original problem into as many number of sub-problems as the variables, solves each sub-problem individually and then obtains the solution of the original problem by integrating the solutions of the sub-problems. It is a systematic, complete enumeration technique.

Principles of optimality / BELLMAN'S optimality:-

principle of optimality due to Bellman's

"An optimal policy has the property that whatever the

...the remaining decisions must constitute an optimal policy with respect to the resulting from the first decision".

The principle of optimality implies that given the initial state of a system, an optimal policy for the sub-sequent stages does not depend upon the policy adopted at the preceding stages. That is the effect of a current policy decision on any of the policy decisions of the preceding stages need not be taken into account at all. It is usually referred to as the Markovian property of dynamic programming.

Thursday
5/4/2017

Terminology in Dynamic programming problem

Stage: - Each sub-problem of the original problem is known as a stage.

Alternative m_i : - In a given stage i , there may be more than one choice of carrying out a task. Each choice is known as Alternative m_i .

State variable x_i : A possible value of a resource within its permitted range at a given stage i is known as state variable, x_i .

Recursive function ($f_i(x_i)$): A function which links the measure of performance of interest of the current stage with the cumulative measure of performance of the previous stages / succeeding stages as a function of the state variable of the current stage is known as the Recursive function of the current stage. Let $f_i(x_i) = \max [R(m_i)]$

$$f_i(x_i) = \max \{ R(m_i) + f_{i-1} [x_i - c(m_i)] \} \quad i = 1, 2, 3, \dots, n$$

for feasible m_i where, n is the total no. of stages.

$R(m_i)$ is the measure of performance (like return) due to

alternative m_i of the stage i is the required for the alternative m_i of the stage i . The value of the measure of performance upto current stage i from the stage 1. If the amount of resource allocated upto the current stage x_i when forward Recursion is used.

Best Recursive function value:-

In a stage i , the lowest (minimization problem) highest (maximization problem) value of the Recursive function for a given value of x_i is known as the best recursive function value.

Best Alternative in a given stage i :-

In a given stage i , the alternative corresponding to the best recursive function value for a given value of x_i is known as the Best Alternative for that value of x_i .

Backward Recursive function:-

Here computation begins from the last stages/ Sub-problem, and this stage will be numbered as stage one while the first Sub-problem will be numbered as the last stage. Since the Recursion proceeds in a Backward direction, this type of Recursive function is known as Backward Recursive function.

Forward Recursive function:-

While defining the stages of the original problem, the first Sub-problem will be numbered as Stage one and last Sub-problem will be numbered as the last stage. Then the Recursive function will be defined as per this Assumption. This type of Recursive function is known as Forward Recursive function.

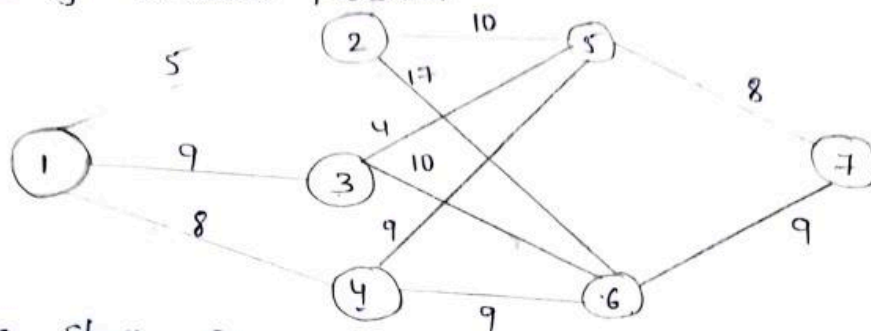
The dynamic programming can be applied to many real life situations. A sample list of applications of the dynamic programming are

1. Capital Budgeting problem.
2. Reliability improvement problem.
3. Stage-coach problem or shortest-path problem.
4. Cargo loading problem.
5. Minimising total Tardiness in Single machine Scheduling problem.
6. Optimal Sub-Dividing problem.
7. Linear programming problem.

Stage-coach problem or shortest-path problem:-

Stage-coach problem is a shortest path problem in which the objective is to find the shortest distance and the corresponding path from a given source node to a given destination node in a given distance network.

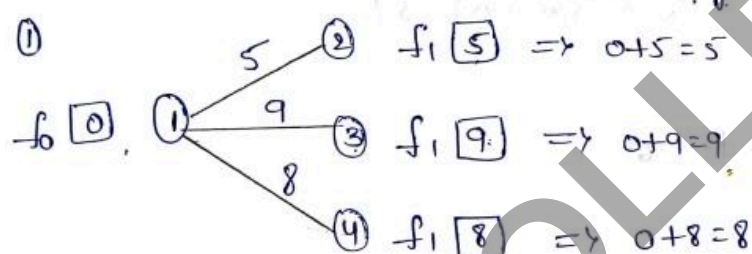
It gives the shortest distance and the path from origin to destination in a given distance Network Problem. Consider the following distance of Network problem.



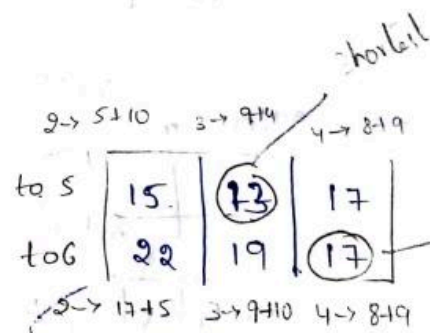
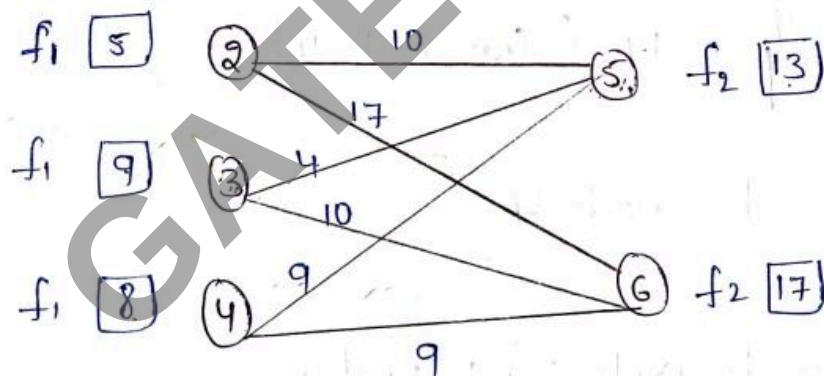
Sol We shall find the path distance that connects node 1 and node 7. To this we divide the given Network into different stages as follows.

The given problem is three stage problem

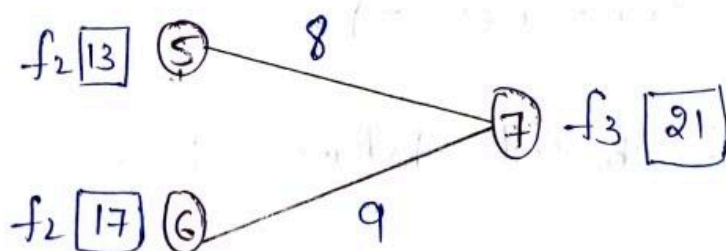
stage ①



stage ②



stage ③



$$f_0(1) = 0$$

stage ① - shortest distance from node ① to node ② = 5 = $f_1(2)$

shortest distance from node ① to node ③ = $f_1(3) = 9$

shortest distance from node ① to node ④ = $f_1(4) = 8$

stage ② - Node ⑤ is reached from nodes ②, ③, ④. Therefore shortest distance to node ⑤ is

$$\min_{i=2,3,4} \left\{ (\text{shortest distance to node } i) + \text{distance from node } i \text{ to node } 5 \right\}$$

$$\Rightarrow \min \left\{ \begin{array}{l} 5+10 = 15 \\ 9+4 = 13 \\ 8+9 = 17 \end{array} \right\} = 13 \text{ (from node 3)}$$

shortest distance to node ⑥

$$\min_{i=2,3,4} \left\{ (\text{shortest distance to node } i) + \text{distance from node } i \text{ to node } 6 \right\}$$

$$\Rightarrow \min \left\{ \begin{array}{l} 5+17 = 22 \\ 9+10 = 19 \\ 8+9 = 17 \end{array} \right\} = 17 \text{ (from node 4)}$$

stage ③ - shortest distance to node 7

$$\min_{i=5,6} \left\{ (\text{shortest distance to node } i) + \text{distance from node } i \text{ to node } 7 \right\}$$

$$\Rightarrow \min \left\{ \begin{array}{l} 13+8 = 21 \\ 17+9 = 26 \end{array} \right\} = 21 \text{ (from node 5)}$$

from the above calculation the shortest distance between node 1 and node 7 is 21 km for the corresponding path, we see from stage 3

node 7 is linked to node 5, in stage 2 node 5 is linked to node 3, in stage 1 node 3 is linked to node 1

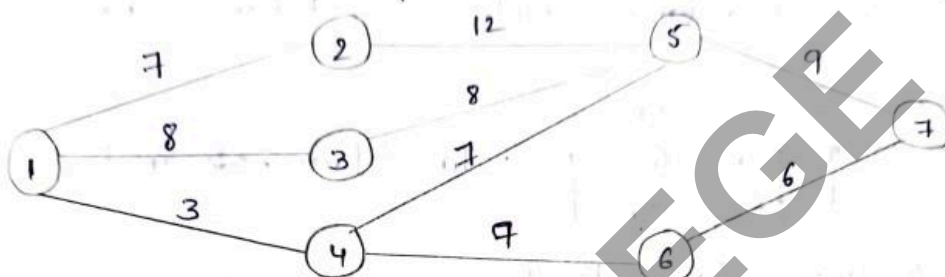
optimum path is 1 to 3, 3 to 5, 5 to 7

$$\Rightarrow 9 + 4 + 8 = 21$$



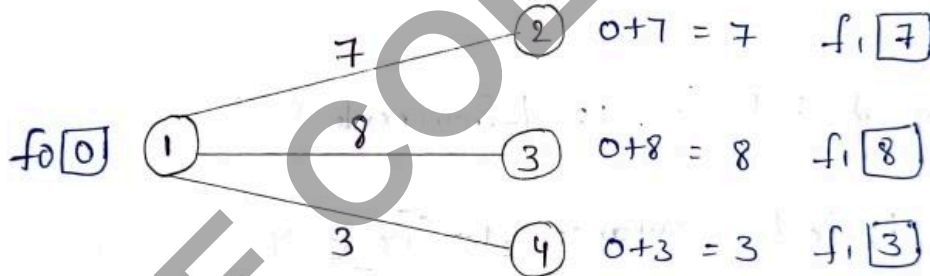
Friday
6/4/2018

2) Find the shortest distance and path for the following data/Network

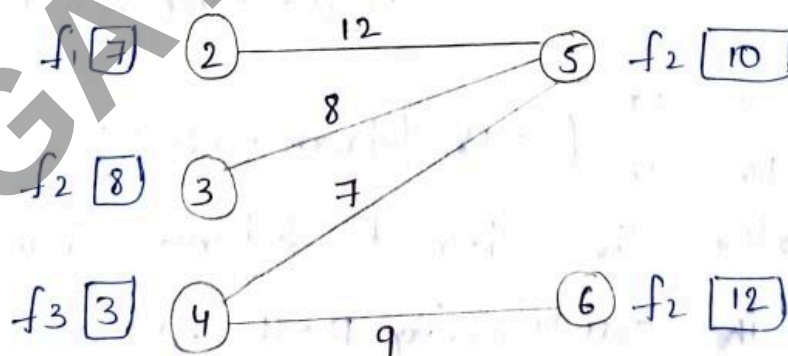


Sol The given problem is three stage problem

Stage ①

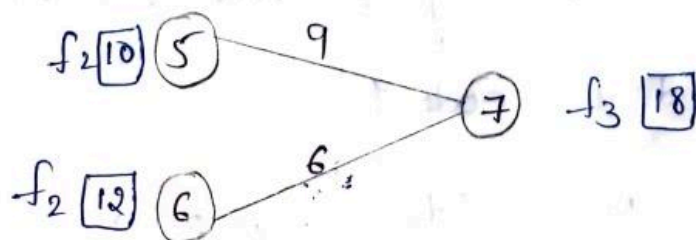


Stage ②



7+12	8+8	3+7
19	16	10
3+7=10		3+9=12

Stage ③



$$f_0(1) = 0$$

Stage ① - shortest distance from node 1 to node 2 = 7 = $f_1(2)$
 shortest distance from node 1 to node 3 = $f_1(3) = 8$
 shortest distance from node 1 to node 4 = $f_1(4) = 3$

Stage ② - node 5 is reached from nodes 2, 3, 4. Therefore shortest distance to node 5 is

$$\min_{i=2,3,4} \{ (\text{shortest distance to node } i) + (\text{distance from node } i \text{ to node } 5) \}$$

$$\Rightarrow \min \begin{cases} 7+2 = 9 \\ 8+8 = 16 \\ 3+7 = 10 \end{cases} = 10 \text{ (from node 4)}$$

shortest distance to node 6

$$\min_{i=2,3,4} \{ (\text{shortest distance to node } i) + (\text{distance from node } i \text{ to node } 6) \}$$

$$\Rightarrow \min \{ 3+9 \} = 12 \text{ (from node 4)}$$

Stage ③ - shortest distance to node 7

$$\min_{i=5,6} \{ (\text{shortest distance to node } i) + (\text{distance from node } i \text{ to node } 7) \}$$

$$\Rightarrow \min \begin{cases} 10+9 = 19 \\ 12+6 = 18 \end{cases} = 18 \text{ (from node 6)}$$

from the above calculation the shortest distance from node 1 to node 7 is 18 km for the corresponding path, we see from stage ③

node 7 is linked to node 6, In stage-2 node 6 is linked to node 4, In stage-1 node 4 is linked to node 1.

Optimum path is 1 to 4, 4 to 6, 6 to 7 $\Rightarrow 3+9+6 = 18$

Backward Recursive Approach problem

① Consider the first problem equations which is given as follows.

	2	3	4		5	6		7
1	5	9	8	2	10	17	3	4
				3	4	10	4	9
				4	9	9		
							5	8
							6	9

Sol

$$f_4(x_4) = 0 \quad ; \quad \text{for } x_4 = 7$$

Since it is three stage problem

Stage 3 - node 7 is connected to node 5 and node 6. Therefore stage-3 results as follows.

State variable x_3	Alternative m_3		Optimal solution	
	$x_4 = 7$		$f_3(x_3)$	m_3^*
5	8		8	7
6	9		9	7

Stage-2 - Node 5 and Node 6 are connected to node 2, 3, 4 therefore stage-2 results are as follows

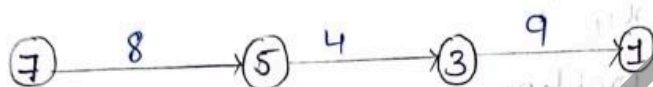
State variable x_2	Alternative m_2		Optimal solution	
	$x_3 = 5$	$x_3 = 6$	$f_2(x_2)$	m_2^*
2	$10+8 = 18$	$17+9 = 26$	$\min(18, 26) = 18$	5
3	$4+8 = 12$	$10+9 = 19$	$\min(12, 19) = 12$	5
4	$9+8 = 17$	$9+9 = 18$	$\min(17, 18) = 17$	5

Stage 1 $\rightarrow$ Node 2, 3, 4 are connected to Node 1. therefore stage 1 results are as follows.

state variable x_1	Alternative m_1			optimal solution	
	$x_2 = 2$	$x_2 = 3$	$x_2 = 4$	$f_1(x_1)$	m_1^*
1	$5+18=23$	$9+12=21$	$8+17=25$	$\min(23, 21, 25)=21$	3

The minimum distance = 21 km

Shortest path



2) Solve the following stage-coach problem using Backward Recursive function

function

	B	C	D
A	2	4	3

	E	F	G
B	7	4	6
C	3	2	4
D	4	1	5

	H	I
E	1	4
F	6	3
G	3	3

	J
H	3
I	4

$f_5(x_5) = 0$; for $x_5 = J$.
Since it is four stage problem.

Stage 4 - Node J is connected to Node H and Node I. Therefore stage-4 results as follows.

State variable x_4	Alternative m_4		optimal solution	
	$x_5 = J$		$f_4(x_4)$	m_4^*
H	3		3	J
I	4		4	J

Stage 3 - Node H and Node I are connected to nodes E, F, G. Therefore stage 3 results as follows

State variable x_3	Alternative m_1 $x_4 = 11$	Alternative m_2 $x_4 = 7$	Alternative m_3 $x_4 = 8$	Optimal solution m_3^*
E	$1+3=4$	$4+4=8$	$\min(4,8)=4$	E
F	$6+3=9$	$3+4=7$	$\min(9,7)=7$	F
G	$3+3=6$	$3+4=7$	$\min(6,7)=6$	H

Stage 2 - Nodes E, F, G are connected to Nodes B, C, D. Therefore stage 2 results as follows.

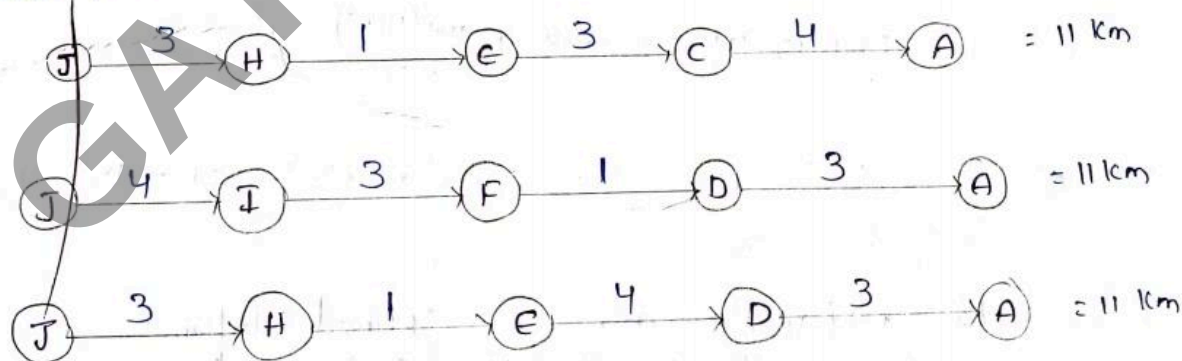
State variable x_2	Alternative m_2			Optimal solution	
	$x_3 = E$	$x_3 = F$	$x_3 = G$	$f_2(x_2)$	m_2^*
B	$7+4=11$	$4+7=11$	$6+6=12$	$\min(11,11,12)=11$	E, F
C	$3+4=7$	$2+7=9$	$4+6=10$	$\min(7,9,10)=7$	E
D	$4+4=8$	$1+7=8$	$5+6=11$	$\min(8,8,11)=8$	E, F

Stage 1 - The nodes B, C, D are connected to the node A. Therefore the stage 1 results as follows.

State variable x_1	Alternative m_1			Optimal solution	
	$x_2 = B$	$x_2 = C$	$x_2 = D$	$f_1(x_1)$	m_1^*
A	$2+11=13$	$3+8=11$	$4+7=11$	$\min(13,11,11)=11$	C, D

The minimum Distance = 11 km

Shortest path



3) Solve the following stage reach problem using forward method

	B	C	D	E	F	G	H	I	J
A	7	3	4	7	4	6	1	8	11
				5	2	4	6	9	2
				6	1	5	3	3	1

Sol

$f_0(x_0) = 0$ for $x_0 = A$

Since it is a four stage problem

Stage 1:- Node A is connected to Nodes B, C and D. Therefore stage 1 results as follows

State variable x_1	Alternative m_1 $x_0 = A$	Optimal solution $f_1(x_1)$	m_1^*
B	7	7	A
C	3	3	A
D	4	4	A

Stage 2:- Nodes B, C, D are connected to Nodes E, F, G. Therefore stage 2 results as follows

State variable x_2	Alternative m_2 $x_1 = B, x_1 = C, x_1 = D$			Optimal solution $f_2(x_2)$	m_2^*
E	$7+7=14$	$5+3=8$	$6+4=10$	$\min(14, 8, 10) = 8$	C
F	$6+7=11$	$2+3=5$	$1+4=5$	$\min(11, 5, 5) = 5$	C, D
G	$6+7=13$	$4+3=7$	$5+4=9$	$\min(13, 7, 9) = 7$	C

Stage 3:- Nodes E, F, G are connected to Nodes H and I. Therefore stage 3 results as follows

State variable x_3	Alternative m_3 $x_2 = E, x_2 = F, x_2 = G$			Optimal solution $f_3(x_3)$	m_3^*
H	$1+8=9$	$6+5=11$	$3+7=10$	$\min(9, 11, 10) = 9$	E
I	$8+8=16$	$9+5=14$	$3+7=10$	$\min(16, 14, 10) = 10$	G

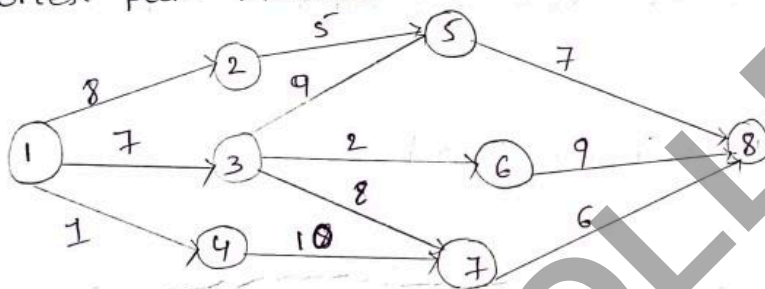
state variable x_4	Alternative m_4 $x_3 = H$	Alternative m_4 $x_3 = I$	optimal solution $f_4(x_4)$	optimal solution m_4
J	$2 + 9 = 11$	$1 + 10 = 11$	$\min(11, 11) = 11$	H, I

Shortest path.

$$(A) \xrightarrow{3} (C) \xrightarrow{5} (E) \xrightarrow{1} (H) \xrightarrow{2} (J) \Rightarrow 3+5+1+2 = 11 \text{ km}$$

$$(A) \xrightarrow{3} (C) \xrightarrow{4} (G) \xrightarrow{3} (I) \xrightarrow{1} (J) \Rightarrow 3+4+3+1 = 11 \text{ km}$$

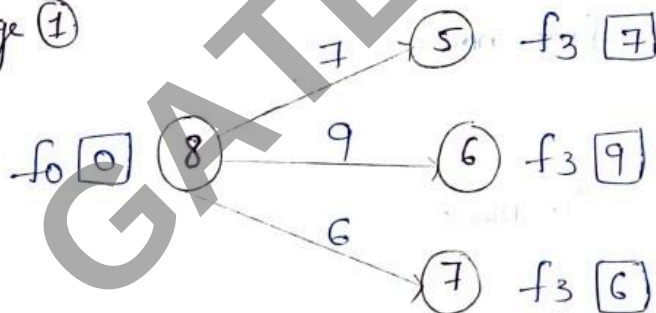
4) Solve the following Network using stage coach method (or) shortest path method.



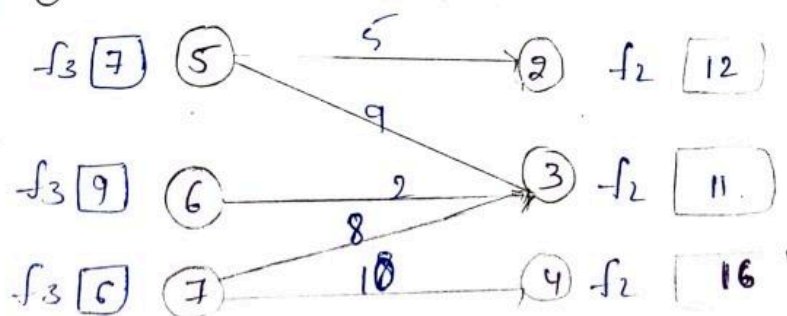
Sol Backward Recursive Approach:-

The given problem is ^{Three} four stage problem

stage ①



stage ②

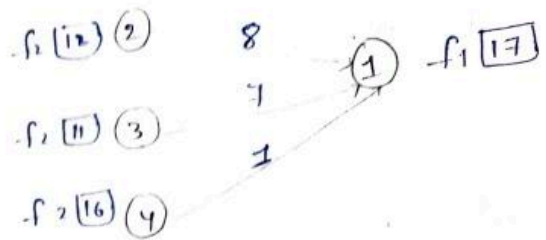
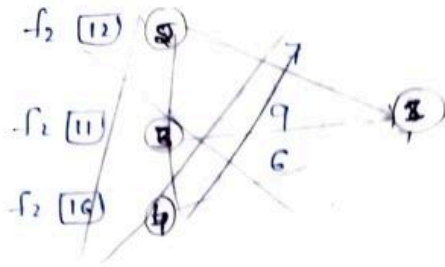


12 ✓

16
11
14
16

11
11
14
16

Stage 3:-



Stage 0:- The shortest path distance from node 8 to node 8.
 $f_0(8) = 0$

Stage 1:- The shortest distance from node 8 to node 5, 6, 7;
 $f_3(5) = 7$
 The shortest distance from node 8 to node 6;
 $f_3(6) = 9$
 The shortest distance from node 8 to node 7;
 $f_3(7) = 6$

Stage 2:- Node 5 is reached to node 2, 3. Therefore shortest distance to node 5 is

$$\Rightarrow \min \begin{Bmatrix} 7+5 \\ 7+9 \end{Bmatrix} = 12 \text{ (from node 2)}$$

Shortest distance to node 6

node 6 is reached to node 3.

$$\min \{ 9+2 \} = 11 \text{ (from node 3)}$$

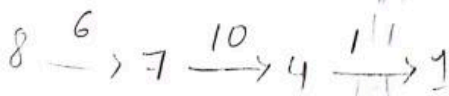
node 7 is reached to node 4

$$\min \begin{Bmatrix} 6+8 = 14 \\ 6+10 = 16 \end{Bmatrix} = 14 \text{ from node 3}$$

Stage 3:-

node 2, 3, 4 is reached to node 1

$$\min \begin{Bmatrix} 12+8 = 20 \\ 11+7 = 18 \\ 16+1 = 17 \end{Bmatrix} = 17 \text{ (from node 4)}$$



Consider the general linear programming problem

$$\text{Max } z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

Sub to Constraints:

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \leq b_i$$

where $i = 1, 2, 3, \dots, m$ &

$$x_j \geq 0$$

$$j = 1, 2, 3, \dots, n$$

GATE COLLEGE

And this problem can be formulated as a dynamic programming problem as follows:

Let the general LPP be considered as a multi-stage problem with each activity 'j' ($j = 1, 2, 3, \dots, m$) as individual stage. Then this is an n stage problem and the decision variables (alternatives) are the levels of activities $x_j \geq 0$. At stage 'j', an x_j is continues to each activity has an infinite number of alternatives within the feasible region. We know that allocation problems are the particular types of LPP. These problems requires the allocation of available resources to the activities. Each constraints represents the limitation of different resources and $b_1, b_2, \dots, b_m$ are the amounts of available resources. Since there are 'm' resources, the stage must be represented by an 'm' component vector.

$$S = (b_1, b_2, b_3, \dots, b_m)$$

Let $f_n(b_1, b_2, \dots, b_m)$ be the maximum value of the general LPP define about for stages $x_1, x_2, \dots, x_m$ for states $b_1, b_2, \dots, b_m$ using forward computational procedure the recursive equation is $f_n(b_1, b_2, \dots, b_m)$

$$= \max_{0 \leq x_j \leq b_j} \{ C_j x_j + f_{n-1}(b_1 - a_{1j} x_j, b_2 - a_{2j} x_j, \dots, b_m - a_{mj} x_j) \}$$

$$0 \leq x_j \leq b_j$$

The minimum value of b can be assumed as $b = \min \left\{ \frac{b_1}{a_{1j}}, \frac{b_2}{a_{2j}}, \frac{b_3}{a_{3j}}, \dots, \frac{b_m}{a_{mj}} \right\}$ because the

minimum value satisfies the set of constraints simultaneously.

Solve the following APP by using DPP

$$\max Z = 3x_1 + 4x_2$$

$$\text{S.P.C} \quad 2x_1 + 5x_2 \leq 120$$

$$2x_1 + x_2 \leq 40$$

$$\text{NNR} \quad x_1, x_2 \geq 0$$

given problem divided into stages - no of sub-problem.
states - no of alternative possible ways
Alternatives

Sol Given problem is a two stage problem since it consists of two decision variables x_1 & x_2 . We use backward recursive equation method to solve the problem.

$$\max Z = 3x_1 + 4x_2$$

$$\text{S.P.C} \quad \begin{matrix} b_{11} & b_{12} \\ 2x_1 & + 5x_2 \end{matrix} \leq 120$$

$$\begin{matrix} b_{21} & b_{22} \\ 2x_1 & + x_2 \end{matrix} \leq 40$$

$$\text{NNR} \quad x_1, x_2 \geq 0$$

Stage : 2

$$f_2(x_1, x_2) = \max(4x_2) = 4 \max x_2$$

from constraints:

$$2x_1 + 5x_2 \leq 120$$

$$x_2 \leq \frac{120 - 2x_1}{5}$$

$$(2) \Rightarrow 2x_1 + x_2 \leq 40$$

$$x_2 \leq 40 - 2x_1$$

Condition:

Max of x_2 is the minimum of the above two Equations that is

$$x_2 \leq \min \left\{ \frac{120 - 2x_1}{5}, 40 - 2x_1 \right\}$$

Stage : (1)

$$f_1(b_1, b_2) = \max\{4x_1\} + \max\{4x_2\}$$

$$= \max\left\{ 8x_1 + 4 \min \left\{ \frac{120 - 2x_1}{5}, 40 - 2x_1 \right\} \right\}$$

$$= \max \left\{ \begin{array}{l} 3x_1 + 4 \left(\frac{120 - 2x_1}{5} \right) \text{ if } \frac{120 - 2x_1}{5} \leq 40 - 2x_1 \\ 3x_1 + 4(40 - 2x_1) \text{ if } \frac{120 - 2x_1}{5} \geq 40 - 2x_1 \end{array} \right\}$$

Consider.

$\frac{120 - 2x_1}{5} \leq 40 - 2x_1$	$\frac{120 - 2x_1}{5} \geq 40 - 2x_1$
$120 - 2x_1 \leq 4(50 - 2x_1)$	$120 - 2x_1 \geq 5(40 - 2x_1)$
$x_1 \leq 10$	$x_1 \geq 10$

$$x_2 \leq \min \left\{ \frac{120-2x_1}{5}, 40-2x_1 \right\}$$

$$x_2 \leq \min \left\{ \frac{120-2(10)}{5}, 40-2(10) \right\}$$

$$x_2 \leq \min \{20, 20\}$$

$$x_1 = 10, x_2 = 20$$

$$\max z = 3x_1 + 4x_2$$

$$= 3(10) + 4(20) = 110$$

Solve the following LPP by using DPP

$$\max z = 10x_1 + 30x_2$$

S.P.C

$$3x_1 + 6x_2 \leq 168$$

$$12x_2 \leq 240$$

NNR

$$x_1, x_2 \geq 0$$

Given problem is a two stage problem since it consists of two decision variables x_1 & x_2 . We use backward recursive method to solve the problem.

$$\max z = 10x_1 + 30x_2$$

S.P.C

$$3x_1 + 6x_2 \leq 168$$

$$12x_2 \leq 240$$

NNR

$$x_1, x_2 \geq 0$$

stage : 2 $f(b_{12}, b_{22}) = \max \{c_1 x_1\}$
 $= \max \{10x_1\}$

from constraints

$$\textcircled{1} \quad 3x_1 + 6x_2 \leq 168$$

$$6x_2 \leq 168 - 3x_1$$

$$x_2 \leq \frac{168 - 3x_1}{6}$$

from second constraint

$$12x_2 \leq 240$$

$$x_2 \leq \frac{240}{12}$$

$$x_2 \leq 20$$

Max of x_2 is the minimum of the above two equations that is

$$x_2 \leq \min \left\{ \frac{168 - 3x_1}{6}, 20 \right\}$$

stage : 1

$$f_1(b_{11}, b_{21}) = \max \{c_1 x_1\} + \max \{c_2 x_2\}$$

$$= \max \left\{ 10x_1 + 30 \min \left\{ \frac{168 - 3x_1}{6}, 20 \right\} \right\}$$

$$= \max \left\{ \begin{array}{ll} 10x_1 + 30 \min \left\{ \frac{168 - 3x_1}{6} \right\} & \text{if } \frac{168 - 3x_1}{6} \leq 20 \\ 10x_1 + 30(20) & \text{if } 20 \leq \frac{168 - 3x_1}{6} \end{array} \right\}$$

$$\frac{168 - 3x_1}{8} \leq 20.$$

$$168 - 3x_1 \leq 160$$

$$-3x_1 \leq 160 - 168.$$

$$-3x_1 \leq -8$$

$$x_1 \leq 16.$$

$$\frac{168 - 3x_1}{6} \geq 20$$

$$168 - 3x_1 \geq 120$$

$$x_1 \geq 16.$$

if $x_1 \geq 16$ & $x_1 \leq 16$ then $x_1 = 16.$

$$x_2 \leq \min \left\{ \frac{168 - 3x_1}{6}, 20 \right\}$$

$$x_2 \leq \min \left\{ \frac{168 - 3(16)}{6}, 20 \right\}$$

$$x_2 \leq \min \{ 20, 20 \}$$

$$x_2 \leq 20$$

$$x_1 = 16, \quad x_2 = 20.$$

$$\max z = 10x_1 + 30x_2$$

$$= 10(16) + 30(20)$$

$$= 160 + 600$$

$$= 760$$

solve the LPP by using DP

$$\max z = 2x_1 + 5x_2$$

S.P.C
=

$$2x_1 + x_2 \leq 43$$

$$2x_2 \leq 46$$

N.R

$$x_1, x_2 \geq 0$$

Sol the given problem is two stage problem. Since it consists of two decision variable i.e., x_1 & x_2 . We use forward recursive function to solve the problem.

$$\begin{aligned} \text{Stage:1 } f_1(b_1, b_2) &= \max \{C_1 x_1\} \\ &= \max 2x_1 \\ &= 2 \max x_1 \end{aligned}$$

from the constraints

$$\textcircled{1} \Rightarrow 2x_1 + x_2 \leq 43$$

$$x_1 \leq \frac{43 - x_2}{2}$$

$$\textcircled{2} \Rightarrow x_2 \leq 23$$

Cond max of x_1 is the minimum of above two equations that is

$$\max x_1 \leq \min \left\{ \frac{43 - x_2}{2}, 0 \right\}$$

$$\text{Stage:2 } \max \{C_1 x_1\} + \max \{C_2 x_2\}$$

$$= 2 \min \left\{ \frac{43 - x_2}{2} \right\} + 5x_2$$

$$x_1 \leq \frac{43 - x_2}{2}$$

$$x_1 \leq \frac{43 - 23}{2}$$

$$x_1 \leq \frac{20}{2}$$

$$x_1 \leq 10$$

$$x_1 = 10, \quad x_2 = 23$$

$$\max z = 2x_1 + 5x_2$$

$$= 2(10) + 5(23)$$

$$= 20 + 115$$

$$\max z = 135$$

recursive function.

$$\text{Max } Z = 3x_1 + 4x_2$$

S.R.C

$$2x_1 + 5x_2 \leq 120$$

$$2x_1 + x_2 \leq 40$$

N.N.B

$$x_1, x_2 \geq 0$$

Sol

The given problem is a two stage problem. Since it consists of two decision variables x_1 & x_2 we use forward recursive equation method to solve the problem.

$$\text{Stage: 1 } f_1(b_1, b_2) = \max (C_1 x_1) \\ = 3 \max x_1$$

from constraints:

$$\textcircled{1} \quad 2x_1 + 5x_2 \leq 120$$

$$x_1 \leq \frac{120 - 5x_2}{2} \quad \textcircled{1}$$

$$\textcircled{2} \Rightarrow 2x_1 + x_2 \leq 40$$

$$x_1 \leq \frac{40 - x_2}{2} \quad \textcircled{2}$$

Condition:

Max of x_1 is minimum of above two Equations that is,

$$x_1 \leq \left\{ \frac{120-5x_2}{2}, \frac{40-x_2}{2} \right\}$$

stage :2

$$f_2(b_{12}, b_{22}) = \max \left\{ \min \left\{ \frac{120-5x_2}{2}, \frac{40-x_2}{2} \right\} + 4x_2 \right\}$$

$$= \max \left\{ \begin{array}{l} 3 \left(\frac{120-5x_2}{2} \right) + 4x_2 ; \text{ if } \frac{120-5x_2}{2} \leq \frac{40-x_2}{2} \\ 3 \left(\frac{40-x_2}{2} \right) + 4x_2 ; \text{ if } \frac{120-5x_2}{2} \geq \frac{40-x_2}{2} \end{array} \right\}$$

Consider

$$\frac{120-5x_2}{2} \leq \frac{40-x_2}{2}$$

$$120-5x_2+x_2 \leq 40-120$$

$$-4x_2 \leq -80$$

$$x_2 \leq 20$$

$$\frac{120-5x_2}{2} \geq \frac{40-x_2}{2}$$

$$120-5x_2 \geq 40-x_2$$

$$x_2 \geq 20$$

$$\therefore x_2 \leq 20, \text{ or } x_2 \geq 20 \quad \therefore x_2 = 20$$

$$x_1 \leq \min \left\{ \frac{120-5x_2}{2}, \frac{40-x_2}{2} \right\}$$

$$\leq \min \left\{ \frac{120-5(20)}{2}, \frac{40-20}{2} \right\}$$

$$\leq \min \{10, 10\}$$

$$x_1 = 10$$

$$\max z = 3x_1 + 4x_2$$

$$= 3(10) + 4(20)$$

$$z = 110$$

Use DPP to solve LPP using forward recursive eq. method:

$$\max z = 3x_1 + 5x_2$$

S.T.C

$$x_1 \leq 4$$

$$x_2 \leq 6$$

$$3x_1 + 2x_2 \leq 18$$

MIN

$$x_1, x_2 \geq 0$$

The given problem is a two stage problem. Since it consists of two decision variables x_1 & x_2 . We use forward recursive equation method to solve the problem.

Stage : 1 $f_1(b_1, b_2, b_3) = \max G(x_1)$
 $= 3 \max x_1$

from Constraints

$$\textcircled{1} x_1 \leq 4$$

$$\textcircled{3} 3x_1 + 2x_2 \leq 18$$

$$3x_1 \leq 18 - 2x_2$$

$$x_1 \leq \frac{18 - 2x_2}{3}$$

Max of x_1 is minimum of above two Equations

that is

$$x_1 \leq \min \left\{ 4, \frac{18 - 2x_2}{3} \right\}$$

$$z(b_{12}, b_{22}, b_{32}) = \max_3 \left\{ \min \left\{ 4, \frac{18-2x_2}{3} \right\} + 5x_2 \right.$$

$$= \max \left\{ 3(4) + 5x_2; \text{ if } 4 \leq \frac{18-2x_2}{3} \right. \\ \left. 3\left(\frac{18-2x_2}{3}\right) + 5x_2; \text{ if } \frac{18-2x_2}{3} \leq 4 \right\}$$

Consider

$$4 \leq \frac{18-2x_2}{3}$$

$$12 \leq 18 - 2x_2$$

$$2x_2 \leq 6$$

$$x_2 \leq 3.$$

$$\frac{18-2x_2}{3} \leq 4$$

$$18 - 2x_2 \leq 12$$

$$18 - 12 \leq 2x_2$$

$$x_2 \geq 3.$$

$\therefore x_2 \leq 3$ or $x_2 \geq 3$ that is $x_2 = 3$

$$x_1 \leq \min \left\{ 4, \frac{18-2x_2}{3} \right\}$$

$$x_1 \leq \min \left\{ 4, \frac{18-2(3)}{3} \right\}$$

$$x_1 \leq \min \{4, 4\}$$

$$x_1 = 4$$

$$\max z = 3x_1 + 5x_2$$

$$= 3(4) + 5(3)$$

$$= 12 + 15$$

$$= 27$$

$$\max z = 27$$

A Capital Budgeting problem is a problem in which a given amount of capital is allocated to a set of plants by selecting the most promising alternative for each selected plant such that the total revenue of the organisation is maximised. This is demonstrated using a numerical problem.

GATE COLLEGE

An organisation is planning to diversify its business with a maximum outlay of Rs. 5,00,00,000. It has identified three different locations to install plants. The organisation can invest in one or more of these plants subject to the availability of fund. The different possible alternatives and their investment (in crores of rupees) and present worth of returns during the useful life (in crores of rupees) of each of these plants are summarised in problem. The first row of following problem has zero cost and zero return for all the plants. Hence it is known as Do nothing alternative. Find the optimal allocation of the capital to different plants which will maximise the corresponding sum of the present worth of returns.

Alternative	Plant 1		Plant 2		Plant 3	
	cost	Return	cost	Return	cost	Return
1	0	0	0	0	0	0
2	1	15	2	14	1	3
3	2	18	3	18	2	7
4	4	28	4	21	—	—

Sol Maximum capital amount $C = \text{Rs. } 5,00,00,000/-$. Each plant is treated as a stage. So the number of stages = 3. The plants 1, 2 & 3 are defined as stage 1, stage 2 and stage 3 respectively. So the forward Recursive function is used for this problem.

Stage 1:-

The Recursive function for a given combination of the state variable, x_1 and Alternative m , in the first stage. The corresponding returns are summarised in following table. For each value of the State variables, the best return and the corresponding alternatives are presented in the last two columns respectively.

$$f_1(x_1) = R(m_1)$$

Alternative m_1

state variables	1		2		3		4		$f_1^*(x_1)$	m_1^*
	C	R	C	R	C	R	C	R		
x_1	0	0	1	15	2	18	4	28		
0	0	—	—	—	—	—	—	—	0	1
1	0	15	—	—	—	—	—	—	15	2
2	0	15	18	—	—	—	—	—	18	3
3	0	15	18	—	—	—	—	—	18	3
4	0	15	18	—	—	—	28	—	28	4
5	0	15	18	—	—	—	28	—	28	4

C = cost, R = Return

Stage 2:-

The Recursive function $f_2(x_2)$ for a given combination of the state variable x_2 and Alternative m_2 . In the second stage is given by

$$f_2(x_2) = R(m_2) + f_1(x_2 - c(m_2))$$

The corresponding returns are summarized in the following table. For each value of the state variable, the best return and the corresponding alternatives are presented in the last two columns respectively.

state variable	1		2		3		4		$-f_2(x_2)^*$	m_2^*
	C	R	C	R	C	R	C	R		
x_2	0	0	2	14	3	18	4	21		
0	0								0	1
1	$0+15=15$								15	1
2	$0+18=18$		$14+0=14$						18	1
3	$0+18=18$		$14+15=29$		$18+0=18$				29	2
4	$0+28=28$		$14+18=32$		$18+15=33$		$21+0=21$		33	3
5	$0+28=28$		$14+18=32$		$18+18=36$		$21+15=36$		36	3 and 4

Stage 3:-

The Recursive function $-f_3(x_3)$ for different combinations of state variable x_3 and alternative m_3 in the third stage is $-f_3(x_3)$

$$-f_3(x_3) = R(m_3) + f_2(x_3 - C(m_3))$$

The corresponding returns are summarised in the following table. For each value of the state variable, the best return and the corresponding alternatives are presented in the last two columns respectively.

Stage 3:-

Calculation for stage 3

Alternative m_3

state variable	1		2		3		$-f_3(x_3)^*$	m_3^*
	C	R	C	R	C	R		
x_3	0	0	1	3	2	7		
5	$0+36=36$		$3+33=36$		$7+29=36$		36	1, 2 and 3

The final results of the original problem is traced as in following table. From this table one can visualize the fact that the original problem has

Final Results

stage 3

C^* m_3^*

5 1

stage 2

C^* m_2^*

5-0=5 $\begin{cases} 3 \\ 4 \end{cases}$

stage 1

C^* m_1^*

5-3=2 3
5-4=1 2

operational alternative stage

1 2 3

3 - 3 - 1

2 - 4 - 1

2 - 3 - 2

2 - 2 - 3

5 2

5-1=4 3

4-3=1 2

5 3

5-2=3 2

3-2=1 2

Introduction:-

In many important real world decision making situation it may not be feasible or desirable to reduce^{all} the goals of an organisation into single objective. For example instead of focusing only on optimising profits, the organisation may simultaneously be interested in maintaining a stable work force, increasing its share of market and limiting price increases.

Goal programming is an extension of Linear programming problem or Non-Linear programming Problem involving an objective function with multiple objectives. While developing a goal programming model, the decision variables of the model are to be defined first. Then the managers goals related to the problems are to be listed down and Ranked in order of priority. Since it may be very difficult to rank these goals on a cardinal scale, an ordinal ranking is usually applied to each of the goals. It may not always be possible to fully achieve every goal specified by the decision maker. Thus goal programming is often refer to as a lexicographic procedure in which the various goals are satisfied in order of their relative importance.

Difference between LP and GP Approach

Linear Programming	Goal Programming
1. Single objective (or) one goal.	1. Multiple goals (or) objectives
2. Maximising profit (or) minimising cost.	2. Deviation variables minimised.
3. Optimising	3. Satisfying instead of optimising.
4. Replace slack and surplus variables.	4. Deviation variables are real (d ⁺ d ⁻)

$$\text{Min } Z = P_1 d_1^- + P_2 d_4^+ + (2P_3 d_2^- + P_3 d_3^-) - P_4 d_1^+$$

Sub to constraints:-

$$X_1 + X_2 + d_1^- - d_1^+ = 10$$

$$X_1 + d_2^- = 6$$

$$X_2 + d_3^- = 8$$

$$d_1^+ + d_4^- - d_4^+ = 2$$

where,

$$X_1, X_2, d_1^-, d_1^+, d_2^-, d_3^-, d_4^-, d_4^+ \geq 0$$

where

P_1, P_2, P_3, P_4 are the priority levels from high to low.

$P_1 d_1^-$ is the objective function term for goal 1.

$P_2 d_4^+$ is the objective function term for goal 2.

$2(P_3 d_2^- + P_3 d_3^-)$ is the objective function term for goal 3.

$P_4 d_1^+$ is the objective function term for goal 4.

d_1^+ = over Achievement.

d_1^- = under Achievement.

X_1, X_2 are Decision variables.

Formulation of Linear Goal programming problem:-

following are the major steps in the formulation of Linear Goal programming problem

Step 1:- Identify the Decision variables of the key decision

Step 2:- Formulate all the objectives or goals of the problem. These are generally determined by

i) The Desire of the Decision maker.

ii) Limited resources.

iii) Any other restrictions either explicitly or implicitly placed on the Desire of the Decision maker.

Step 3:- Reduce the number of goals by eliminating a few negligibly important or redundant goals.

Step 4:- Express each goal in the form of constraint equation by introducing a negative and positive deviation variable (denoted by d_i^- & d_i^+) respectively that is

$$G_i : f_i(x_1, x_2, \dots, x_n) - d_i^+ + d_i^- = b_i$$

$$i = 1, 2, 3, \dots, m$$

where d_i^- = Negative deviation from i^{th} goal (under Achievement) and d_i^+ = Positive Deviation from i^{th} goal (over Achievement).

Step 5:- Assign the Goals to priority levels. All absolute goals (i.e., either $d_i^- = 0$ (or) $d_i^+ = 0$) if any exist, are assigned to top priority.

Step 6:- Establish the Achievement function. The priorities associated with each objective, along with each the deviation variables are used to form what we call the achievement function.

GPP formulation Problems

1) A package of food Manufacturers produce two types of products chips and Soda. The unit profit for a packet of chips is ₹ 80/- and of bottle of Soda is ₹ 40/-. The Goal of the plan manager is to earn a total profit of exactly ₹ 640/- in the next week.

LPP formulation

$$\max z = 80x_1 + 40x_2$$

GPP formulation

Objectives are referred to the optimisation of the performance of the decision for the given problem objective function is defined as

$$\text{Minimum } z = d_1^- + d_1^+$$

Subject to here d_1^- : under achievement of goal RS. 640/-

d_1^+ : over achievement of goal RS. 640/-

Subject to constraints

Here chips as x_1 and Soda as x_2 . So we have two decision variables x_1 and x_2

$$80x_1 + 40x_2 + d_1^- - d_1^+ = 640$$

Non - Negativity Restrictions

$$x_1, x_2, d_1^-, d_1^+ \geq 0$$

2) A Manufacture produces two products A and B the unit profit of the product A is ₹70/- and product B is ₹30/- The goal of manufacture is to earn the profit of exactly ₹800/- in the next week. Formulate this problem as Goal programming problem.

Objectives are referred to the optimisation of the Measure of performance. the decision for the given problem objective function is defined as

$$\text{Minimum } z = d_1^- + d_1^+$$

Here d_1^- : under achievement of goal RS. 800/-

d_1^+ : over achievement of goal RS. 800/-

Subject to constraints

Here product A as x_1 and product B as x_2 . So we have two decision variables x_1 and x_2

$$70x_1 + 30x_2 + d_1^- - d_1^+ = 800$$

Non - Negativity Restrictions:-

$$x_1, x_2, d_1^-, d_1^+ \geq 0$$

A firm produces two products say X and Y. Product X sells for a net profit of ₹80 per unit, Y product Y sells for a net profit of ₹40 per unit. The goal of the firm is to earn ₹900 in the next week. Also the management want to achieve sales volume for the two products close to 17 and 15 respectively. Formulate this problem as a Goal programming model.

Let x_1 and x_2 denote the number of units of products X and Y respectively.

The linear programming formulation of the problem is

$$\text{Max } z = 80x_1 + 40x_2$$

S.T.C

$$x_1 \leq 17$$

$$x_2 \leq 15$$

Non-Negativity Restrictions

$$x_1, x_2 \geq 0$$

Since the goal of the firm as pertains to profit attainment with a target established at ₹900/- per week. The constraints of the problem can be stated as

$$80x_1 + 40x_2 = 900 \quad (\text{Profit target goal})$$

$$x_1 \leq 17 \text{ and } x_2 \leq 15 \quad (\text{Sales target goal})$$

The problem can now be formulated as Goal programming model as follows.

$$\text{Min } z = d_1^- + d_1^+ + d_2^- + d_3^-$$

Subject to the constraints

$$80x_1 + 40x_2 + d_1^- - d_1^+ = 900$$

$$x_1 + d_2^- = 17$$

$$x_2 + d_3^- = 15$$

Non-Negativity Restrictions

$$x_1, x_2, d_1^+, d_1^-, d_2^-, d_3^- \geq 0$$

where d_2^- and d_3^- represents under achievements of sales volume for product

x and y respectively. Since sales target goals are given as the maximum possible sales volume, therefore d_2^+ and d_3^+ are not included in the sales-target constraints.

d_1^- = under achievement of the profit goal of Rs. 900/-

d_1^+ = over achievement of the profit goal of Rs. 900/-

Graphical Solution Method for Goal programming model

The Graphical solution method for Goal programming model is similar to the Graphical solution method for linear programming model. In this approach, the feasible solution space (Region) is indicated by goal priorities in such a way that the deviation from the goal with the highest priority is minimised to the fullest extent possible, before the deviation from the next priority goal is minimized.

If Goal constraints are stated only in terms of deviational variables, then such constraints must be re-stated in terms of the real variables, before proceeding with the Graphical solution.

The Algorithm :-

- Step 1 - Graph all system constraints (those not involving deviational variables) and identify the feasible solutions space. If no system constraints exist, then the feasible solutions space (Region) is the first quadrant. If no feasible solutions space exist, there is no solution to the problem.
- Step 2 - Graph the straight lines corresponding to the goal constraints, labelling deviational variables.
- Step 3 - Within the feasible solutions space identified in step (1), determine the point or points that best satisfy the highest priority goal.

Step 4 - Sequentially consider the remaining goals and the point that satisfy them to the greatest extent possible. make sure that a lower priority goal is not achieved by reducing the degree of achievement of higher priority goals.

Problems:-

1) A company produces motorcycle seats. The company has two production lines. The production rate for Line 1 is 30 seats per hour and for Line 2 is 60 seats per hour. The company has entered into a contract to daily supply 1200 seats daily to another company. currently the normal operation period for each line is 8 hours. The production manager of the company is trying to determine the best daily operations hour for the two lines. he has set the priorities to achieve his goals as given below.

P_1 : produce ~~the~~^{and} deliver 1200 seats daily.

P_2 : Limit the daily overtime operation hours of Line 2 to 3 hours.

P_3 : Minimize the under utilization of the regular daily operation hours of each line. Assign differential weights based on the relative productivity rate.

P_4 : Minimize the daily overtime operation hours of each line as much as possible. Assign differential weights based on the relative cost of overtime. It is assumed that the cost of operation is identical for the two production lines.

Formulate this problem as GP model and then solve it by using the Graphical method.

Sol Model formulation:-

Let x_1 and x_2 equal to daily operation hours for the line 1 and line 2 respectively.

It may be noted in the problem that the first criterion for determining the differential weights in the third priority goal is the relative cost of

Overtime. The productivity rates for Line 1 is 50 to 60. Therefore the relative cost resulting from an hour of overtime is greater for Line 1 than for Line 2. The relative cost of overtime ratio for Line 1 to Line 2 will be Line 6 to Line 5. Second the criterion for determining the differential weights in the fourth priority goal is relative productivity rate. Since the productivity of Line 1 is 50 seats per hour and of Line 2 is 60 seats per hour, the productivity goals are weighted for Line 1 to Line 2 as Line 5 to Line 6. The Goal programming model for this problem now be formulated as follows:

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$$\text{Minimize } Z = P_1 d_1^- + P_2 d_4^+ + P_3 (5d_2^- + 6d_3^-) + P_4 (6d_2^+ + 5d_3^+)$$

S.T.C

$$50x_1 + 60x_2 + d_1^- - d_1^+ = 1200$$

$$x_1 + d_2^- - d_2^+ = 8$$

$$x_2 + d_3^- - d_3^+ = 8$$

$$x_2 + d_4^- - d_4^+ = 11$$

Non-Negativity restrictions

$$x_1, x_2, d_i^+, d_i^- \geq 0 \text{ for all } i = 1, 2, 3, 4$$

$$50x_1 + 60x_2 = 1200$$

When $x_1 = 0$ then

$$50(0) + 60x_2 = 1200$$

$$60x_2 = 1200 \Rightarrow x_2 = 20$$

x_1	0	24
x_2	20	0

When $x_2 = 0$ then

$$50x_1 + 60(0) = 1200$$

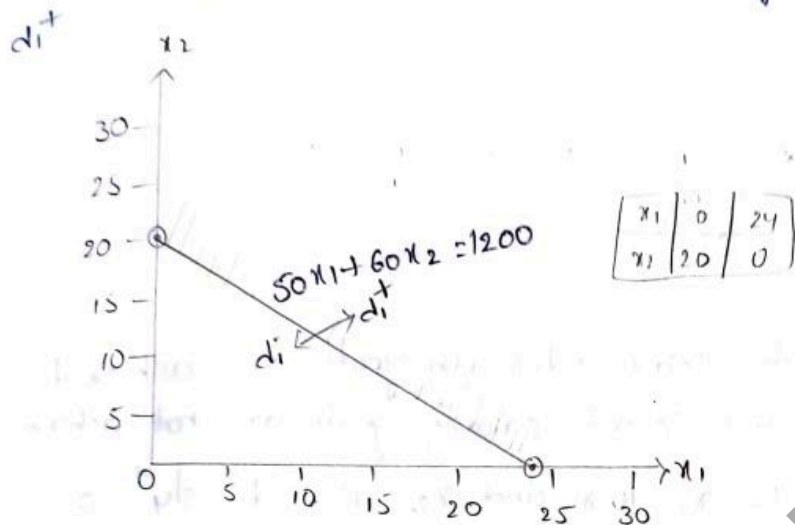
$$50x_1 = 1200 \Rightarrow x_1 = 24$$

$$x_1 + d_2^- - d_2^+ = 8 \Rightarrow x_1 = 8 ; x_2 = 0$$

$$x_2 + d_3^- - d_3^+ = 8 \Rightarrow x_2 = 8 ; x_1 = 0$$

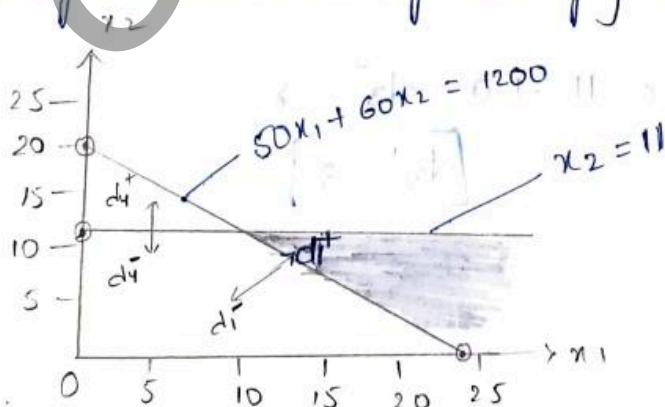
$$x_2 + d_4^- - d_4^+ = 11 \Rightarrow x_2 = 11 ; x_1 = 0$$

To solve this problem, each goal constraint is graphed individually, starting with the one that has the highest priority deviational variable. Since the profit goal constraint deviation d_1^- has priority P_1 in the objective function, this constraint is graphed, ignoring the deviational variables d_1^+ and d_1^- .

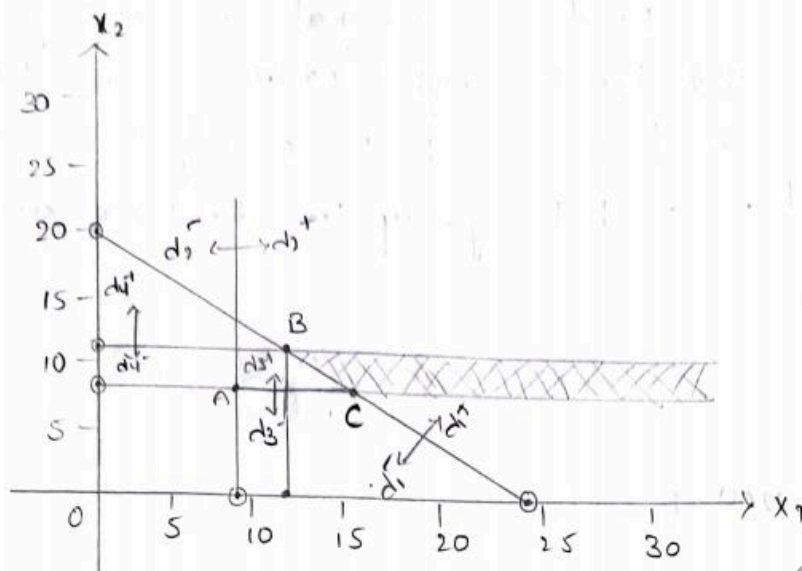


To minimize d_1^- under achievement of delivering 1200 seats, the feasible area is shaded region. Any point in the shaded region satisfies the first goal because the production exceeds 1200 seats.

$\Rightarrow$ Second priority goal is minimizing d_4^+ . The region above the constraint line $x_2 = 11$ represents the value d_4^+ , while the region below the line stands for d_4^- . To avoid the overachievement of the second goal the area above the line is eliminated. But this must be attained within the feasible area that has already been defined by satisfying the first goal.



$\Rightarrow$ The third goal is to avoid under utilisation of regular daily operation hours of each line. This means that both d_2^- and d_3^- should be



At point A in last graph both third and fourth goals are achieved. However at this point the first and second priority goals are not achieved.

At point B however the solution is $x_1 = 10.8$ and $x_2 = 11$, $d_1^- = d_4^+ = 0$

$$d_2^+ = 2.8, d_3^+ = 3, d_3^- = 0, d_2^- = 0$$

$$x_1 + d_2^- - d_2^+ = 8$$

$$10.8 + d_2^- - d_2^+ = 8$$

$$-d_2^+ = 8 - 10.8$$

$$-d_2^+ = -2.8 \Rightarrow d_2^+ = 2.8$$

$$x_2 + d_3^- - d_3^+ = 8 \Rightarrow x_2 + 0 - d_3^+ = 8$$

$$\Rightarrow x_2 + 0 - d_3^+ = 8$$

$$\Rightarrow 11 + 0 - d_3^+ = 8$$

$$x_2 + d_3^- - d_3^+ = 8 \Rightarrow 11 + 0 - d_3^+ = 8$$

$$d_3^+ = 3$$

$$x_2 + d_4^- - d_4^+ = 11$$

$$11 + d_4^- - d_4^+ = 11$$

$$d_4^+ = 0$$

Hence, the 1st, 2nd and 3rd priority goals are completely achieved but the 4th priority goal is not completely achieved. This is because the production line one has 2.8 hrs of overtime ($d_1^+ = 2.8$) and production line 2 has 3 hrs of over-time ($d_2^+ = 3$).

Modified Simplex Method of Goal programming:-

The Simplex method for solving a goal programming is similar to that of an linear programming problem. The features of the Simplex method for the goal programming problem are

Step 1:- The Z_j and $C_j - Z_j$ values are computed separately for each of the ranked goals, $P_1, P_2, P_3, \dots$. This is because different goals are measured in different units. These are shown from bottom to top i.e., first priority goal (P_1) is shown at the bottom and least priority goal at the top.

Step 2:- The optimality criterion Z_j or $C_j - Z_j$ becomes a matrix of $K \times n$ size, where K represents the number of pre-emptive priority levels and 'n' is the number of variables including both Decision and Deviation variables.

Step 3:- First examine $C_j - Z_j$ values in the P_1 -row. If all $C_j - Z_j \leq 0$ at the highest priority level in the same column, then the optimal solution has been obtained.

If $C_j - Z_j \geq 0$, at a certain priority level, and there is no negative entry at higher unachieved priority levels, in the same column, the current solution is not optimal.

Step 4:- If the target value of each goal in X_B -column is zero, the solution is optimal.

Step 5:- To determine the variable to be entered in to the new solution mix, start examining ($C_j - Z_j$) row of highest priority (P_1) and select the

largest Negative value. Otherwise, select the next higher priority (P_2) and select the largest Negative value.

Step 5:- Apply the usual procedure for calculating the minimum ratio to choose a variable that needs to leave the current solution mix (basis).

Step 6:- Any negative value in the $(C_j - Z_j)$ row that has positive $(C_j - Z_j)$ value under any lower priority rows are ignored. This is because that deviations from the highest priority goal would be increased with the entry of this variable in the solution mix.

1) Solve the following goal programming problem

$$\text{Min } Z = 2P_1 d_1^+ + P_2 d_2^+ + P_3 d_1^+$$

S.T.C

$$2x_1 + 2x_2 + d_1^- - d_1^+ = 20$$

$$d_1^+ + d_2^- - d_2^+ = 10$$

N.N.R

$$x_1, x_2, d_1^+, d_1^-, d_2^-, d_2^+ \geq 0$$

Sol Canonical form of GPP :-

$$\text{Min } Z = 2P_1 d_1^- + P_2 d_2^+ + P_3 d_1^+$$

$$\text{Min } Z = 2P_1 d_1^- + P_3 d_1^+ + 0d_2^- + P_2 d_2^+$$

S.T.C

$$2x_1 + 2x_2 + d_1^- - d_1^+ + 0d_2^- + 0d_2^+ = 20$$

$$0x_1 + 0x_2 + 0d_1^- + d_1^+ + d_2^- - d_2^+ = 10$$

Non Negativity Restrictions

$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+ \geq 0$$

Initial Simplex Table

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CB	Basic variable	Cj 0 0 P ₃ 2P ₁ P ₂ 0							Solution	Ratio
		x ₁	x ₂	d ₁ ⁺	d ₁ ⁻	d ₂ ⁺	d ₂ ⁻			
2P ₁	d ₁ ⁻	2	2	-1	1	0	0	20	20/2 = 10	
0	d ₂ ⁻	0	0	1	0	-1	1	10	10/0 = -	
	Zj	4P ₁	4P ₁	-2P ₁	2P ₁	0	0	40P ₁		
	Cj - Zj	-4P ₁	-4P ₁	P ₃ - 2P ₁	0	P ₂	0			
	P ₃	0	0	1	0	0	0	0		
	P ₂	0	0	0	0	1	0	0		
	P ₁	-4	-4	2	0	0	0	40		

↑

x₁ - Entering variableIteration - I d₁⁻ - Leaving variable

CB	Basic variable	Cj 0 0 P ₃ 2P ₁ P ₂ 0							Solution	Ratio
		x ₁	x ₂	d ₁ ⁺	d ₁ ⁻	d ₂ ⁺	d ₂ ⁻			
0	x ₁	1	1	-1/2	1/2	0	0	10		
0	d ₂ ⁻	0	0	1	0	-1	1	10		
	Zj	0	0	0	0	0	0	0		
	Cj - Zj	0	0	P ₃	2P ₁	P ₂	0			
	P ₃	0	0	1	0	0	0	0		
	P ₂	0	0	0	0	1	0	0		
	P ₁	0	0	0	2	0	0	0		

All $C_j - Z_j \geq 0$ Optimality is reached then $x_1 = 10$, $x_2 = 0$, $d_2^- = 10$

$$2x_1 + 2x_2 + d_1^- - d_1^+ + 0d_2^- + 0d_2^+ = 20$$

$$2(10) + 2(0) + 0 - 0 + 0(10) + 0 \cdot 0 = 20$$

$$20 = 20$$

$$\text{Min } Z = P_1 d_1^- + P_2 d_4^+ + (2P_3 d_2^- + P_3 d_3^-) + P_4 d_1^+$$

Subject to constraints

$$x_1 + x_2 + d_1^- - d_1^+ = 10$$

$$x_1 + d_2^- = 6$$

$$x_2 + d_3^- = 8$$

$$d_1^+ + d_4^- - d_4^+ = 2$$

Non-Negativity Restrictions

Sol

$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_3^-, d_4^-, d_4^+ \geq 0$$

In the above problem x_1 and x_2 are Decision variables and the remaining variables are deviational variables. The Initial table for the problem is presented in below.

CB _i	C _j	0	0	P ₁	P ₄	2P ₃	P ₃	0	P ₂	Solution	Ratio
	B.V	x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-	d_4^-	d_4^+		
P ₁	d_1^-	1	1	1	-1	0	0	0	0	10	10
2P ₃	d_2^-	1	0	0	0	1	0	0	0	6	6
P ₃	d_3^-	0	1	0	0	0	1	0	0	8	-
0	d_4^-	0	0	0	1	0	0	1	-1	2	-
Z _j		P ₁ +2P ₃	P ₁ +P ₃	P ₁	-P ₁	2P ₃	P ₃	0	0	10P ₁ +20P ₃	
C _j -Z _j		-P ₁ -2P ₃	-P ₁ -P ₃	0	P ₄ +P ₁	0	0	0	P ₂		
P ₄		0	0	0	1	0	0	0	0	0	
P ₃		-2	-1	0	0	0	0	0	0	20	
P ₂		0	0	0	0	0	0	0	1	0	
P ₁		-1	-1	0	1	0	0	0	0	10	

↑

Z_j values are

$$Z_1 = P_1(1) + 2P_3(1) + (P_3 \times 0) + (0 \times 0) = P_1 + 2P_3$$

$$Z_2 = P_1 - 2P_3 \quad P_1(1) + 2P_3(0) + P_3(1) + 0 \times 0 = P_1 + P_3$$

$$Z_3 = P_1(1) + 2P_3(0) + P_3(0) + 0 \times 0 = P_1$$

$$Z_4 = P_1(-1) + 2P_3(0) + P_3(0) + 0 \times 1 = -P_1$$

$$Z_5 = P_1(0) + 2P_3(1) + P_3(0) + 0 \times 0 = 2P_3$$

$$Z_6 = P_1(0) + 2P_3(0) + P_3(1) + 0 \times 0 = P_3$$

$$Z_7 = P_1(0) + 2P_3(0) + P_3(0) + 0 \times 1 = 0$$

$$Z_8 = P_1(0) + 2P_3(0) + P_3(0) + 0 \times (-1) = 0$$

$$\begin{aligned} \text{solution column} &= 10 \times P_1 + 2P_3 \times 6 + 8 \times P_3 + 0 \times 2 \\ &= 10P_1 + 12P_3 + 8P_3 \\ &= 10P_1 + 20P_3 \end{aligned}$$

for $C_j - Z_j$

$$0 - (P_1 + 2P_3) = -P_1 - 2P_3$$

$$0 - (P_1 + P_3) = -P_1 - P_3$$

For Priorities (P_1, P_2, P_3, P_4 values are)

$$C_1 - Z_1 = -P_1 - 2P_3$$

so priorities are The values of priorities are co-efficients of $C_1 - Z_1$
that is $P_1 = -1, P_2 = 0, P_3 = -2, P_4 = 0$

$$C_2 - Z_2 = -P_1 - P_3$$

$$\text{where } P_1 = -1, P_2 = 0, P_3 = -1, P_4 = 0$$

$$C_3 - Z_3 = 0$$

$$\text{where } P_1 = 0, P_2 = 0, P_3 = 0, P_4 = 0$$

$$C_4 - Z_4 = P_4 + P_1$$

$$\text{where } P_1 = 1, P_2 = 0, P_3 = 0, P_4 = 1$$

$$C_5 - Z_5 = 0$$

$$\text{where } P_1 = 0, P_2 = 0, P_3 = 0, P_4 = 0$$

$$C_6 - Z_6 = 0 \quad ; \quad C_7 - Z_7 = 0$$

$$C_8 - Z_8 = P_2 \quad \text{where } P_1 = 0, P_2 = 1, P_3 = 0, P_4 = 0$$

for Solution column: - $10P_1 + 20P_3$

The priorities values are

$$P_1 = 10, P_2 = 0, P_3 = 20, P_4 = 0$$

Selection of Key column: -

Examine the values in the $C_j - Z_j$ matrix under the solution column. Each positive entry represents the unattained portion of the goal for P_1 , the unattained level is 10 and for P_3 the unattained level is 20. Start starting from the bottom unattained row, find the most negative or least value or minimum value in $C_j - Z_j$ (for minimization problem). For P_1 in $C_j - Z_j$ matrix, the most negative value is -1 under column x_1 and x_2 . To Break the tie, it is necessary to examine $C_j - Z_j$ values in the next highest unattained priority row, that is in row P_3 of $C_j - Z_j$ matrix. The most Negative value is -2 which occurs in column x_1 . Therefore column x_1 is selected as Key column.

To determine key row divide the values under solution column by the respective values in the key column (column x_1) and then select the row with minimum positive ratio. The second row (row d_2^-) is selected as the key row and Intersection element is key element. So x_1 is entering variable and d_2^- is leaving variable. goto Iteration - I

Iteration - I

(1)

C_B	C_j	0	0	P_1	P_4	$2P_3$	P_3	0	P_2		
BV		x_1	x_2	d_1^-	d_1^+	d_2^-	d_3^-	d_4^-	d_4^+	Solution	Ratio
P_1	d_1^-	0	1	1	-1	0	0	0	0	4	4
0	x_1	1	0	0	0	1	0	0	0	6	-
P_3	d_3^-	0	1	0	0	0	1	0	0	8	8
0	d_4^-	0	0	0	1	0	0	1	-1	2	-
Z_j		0	$P_1 + P_3$	P_1	$-P_1$	0	P_3	0	0	$4P_1 + 8P_3$	
$C_j - Z_j$		0	$-P_1 - P_3$	0	$P_4 + P_1$	$2P_3$	0	0	P_2		
P_4		0	0	0	1	0	0	0	0	0	
P_3		0	-1	0	0	2	0	0	0	8	
P_2		0	0	0	0	0	0	0	0	0	
P_1		0	-1	0	1	0	0	0	0	0	

Key element

X_2 - Entering variable

d_1^- - leaving variable

Iteration - II

CB _i	C _j	0	0	P ₁	P ₄	2P ₃	P ₃	0	P ₂	Solution	Ratio
	B.V	X ₁	X ₂	d ₁ ⁻	d ₁ ⁺	d ₂ ⁻	d ₃ ⁻	d ₄ ⁻	d ₄ ⁺		
0	X ₂	0	1	1	-1	0	0	0	0	4	-4
0	X ₁	1	0	0	0	1	0	0	0	6	-
P ₃	d ₃ ⁻	0	0	-1	1	0	1	0	0	4	4
0	d ₄ ⁻	0	0	0	1	0	0	1	-1	2	2
Z _j		0	0	-P ₃	P ₃	0	P ₃	0	0	4P ₃	
C _j -Z _j		0	0	P ₁ +P ₃	P ₄ -P ₃	2P ₃	0	0	P ₂		
P ₄		0	0	0	1	0	0	0	0	0	
P ₃		0	0	1	-1	2	0	0	0	4	
P ₂		0	0	0	0	0	0	0	1	0	
P ₁		0	0	1	0	0	0	0	0	0	

Selection of key column for only unattained rows corresponding row minimum values corresponding column we select as a key column. Here P₁ and P₂, P₄ are attained goals. Only P₃ is unattained goal. So P₃'s corresponding row minimum value is -1. So the corresponding column is selected as key column.

* d₄⁻ - leaving variable

d₁⁺ - entering variable.

CB _i	C _j	P ₁ P ₂ 2P ₃ P ₃ 0 d ₄ ⁺								Solution
		X ₁	X ₂	d ₁ ⁻	d ₁ ⁺	d ₂ ⁻	d ₃ ⁻	d ₄ ⁻	d ₄ ⁺	
0	X ₂	0	1	1	0	0	0	1	-1	6
0	X ₁	1	0	0	0	1	0	0	0	6
P ₃	d ₃ ⁻	0	0	-1	0	0	1	-1	1	2
P ₄	d ₁ ⁺	0	0	0	1	0	0	1	-1	2
Z _j		0	0	-P ₃	P ₄	0	P ₃	-P ₃ +P ₄	P ₃ -P ₄	2P ₃ +2P ₄
C _j -Z _j		0	0	P ₁ +P ₃	0	2P ₃	0	P ₃ -P ₄	P ₂ -P ₃ +P ₄	
P ₄		0	0	0	0	0	0	-1	1	2
P ₃		0	0	1	0	2	0	1	-1	2
P ₂		0	0	0	0	0	0	0	1	0
P ₁		0	0	1	0	0	0	0	0	0

In criterion matrix, P₃ row has -1 for d₄⁺ column. But at a higher level (P₂ level) in the same column(1) one exists. If we want to try to achieve goal₃ (P₃ row), we will have to sacrifice goal₂ (P₂ row). Hence there is no need to introduce the variable d₄⁺ into the basis. Similarly the P₄ row has -1 under d₄⁻ column. But at P₃ row in the same column, one(1) exists. If we try to achieve goal₄ (P₄ row), we will have to sacrifice goal₃ (P₃ row). Hence, it is not necessary to introduce the variable d₄⁻ into the basis.

Since all other values in the criterion matrix are 0 (zero or more than zero), and the problem is to attain minimization, the optimality is reached. Hence the optimal results are $X_1 = 6$; $X_2 = 6$, $d_3^- = 2$, $d_1^+ = 2$

3) Solve the following GPP by using Modified simplex method

$$\text{Min } z = P_1 d_1^+ + P_2 d_2^+ + P_3 d_1^-$$

S.T.C

$$3x_1 + 3x_2 + d_1^- + d_1^+ = 30$$

$$d_1^+ + d_2^- - d_2^+ = 5$$

NNR

$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+ \geq 0$$

CB;	C _j	0	0	P ₃	P ₁	P ₂	0	Solution	Ratio
	B.V	x ₁	x ₂	d ₁ ⁺	d ₁ ⁻	d ₂ ⁺	d ₂ ⁻		
P ₁	d ₁ ⁻	3	3	1	1	0	0	30	10
0	d ₂ ⁻	0	0	1	0	-1	1	5	-
Z _j		3P ₁	3P ₁	P ₁	P ₁	0	0	30P ₁	
C _j -Z _j		-3P ₁	-3P ₁	P ₃ -P ₁	0	P ₂	0		
P ₃		0	0	1	0	0	0	0	
P ₂		0	0	0	0	1	0	0	
P ₁		-3	-3	-1	0	0	0	30	

x₁ - Entering variable, d₁⁻ - leaving variable

CB;	C _j	0	0	P ₃	P ₁	P ₂	0	Solution	Ratio
	B.V	x ₁	x ₂	d ₁ ⁺	d ₁ ⁻	d ₂ ⁺	d ₂ ⁻		
0	x ₁	1	1	1/3	1/3	0	0	10	
0	d ₂ ⁻	0	0	1	0	-1	1	5	
Z _j		0	0	0	0	0	0		
C _j -Z _j		0	0	P ₃	P ₁	P ₂	0		
P ₃		0	0	1	0	0	0		
P ₁		0	0	0	0	1	0		
P ₂		0	0	0	1	0	0		

$$\text{Min } \text{Min } z = P_1 d_1^- + P_2 d_2^- + P_3 d_1^+ \\ = 0$$

S.T.C

$$3x_1 + 3x_2 + d_1^- - d_1^+ = 30$$

$$3(10) + 0 + 0 + 0 = 30$$

$$30 = 30$$

4) Solve the following GPP

$$\text{Min } z = P_1 d_1^- + P_2 (d_2^- + d_3^-) + P_3 d_1^+$$

S.T.C

$$x_1 + x_2 + d_1^- - d_1^+ = 10$$

$$x_1 + d_2^- = 8$$

$$x_2 + d_3^- = 4$$

N.N.R

$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_3^- \geq 0$$

CB _i	C _j	0	0	P ₁	P ₃	2P ₂	P ₂	Solution	Ratio
	B.V	x ₁	x ₂	d ₁ ⁻	d ₁ ⁺	d ₂ ⁻	d ₃ ⁻		
P ₁	d ₁ ⁻	1	1	1	-1	0	0	10	10
2P ₂	d ₂ ⁻	0	1	0	0	1	0	8	8
P ₂	d ₃ ⁻	0	1	0	0	0	1	4	-
Z _j		P ₁ +2P ₂	P ₁ +P ₂	P ₁	-P ₁	2P ₂	P ₂	10P ₁ +20P ₂	
C _j -Z _j		-P ₁ -2P ₂	-P ₁ -P ₂	0	P ₃ +P ₁	0	0		
P ₃		0	0	0	1	0	0	0	
P ₂		-2	-1	0	0	0	0	20	
P ₁		-1	-1	0	1	0	0	10	

x_1 - Entering variable.

d_2^- - leaving variable.

CB _i	C _j	B.V	X ₁	X ₂	d ₁ ⁻	d ₁ ⁺	d ₂ ⁻	d ₃ ⁻	Solution	Ratio
P ₁	d ₁ ⁻		0	1	1	-1	-1	0	2	2
0	X ₁		1	0	0	0	1	0	8	-
P ₂	d ₃ ⁻		0	1	0	0	0	1	4	4
	Z _j		0	P ₁ +P ₂	P ₁	-P ₁	-P ₁	P ₂	2P ₁ +4P ₂	
	C _j -Z _j		0	-P ₁ -P ₂	0	P ₃ +P ₁	2P ₂ +P ₁	0		
P ₃			0	0	0	1	0	0	0	
P ₂			0	-1	0	0	2	0	4	
P ₁			0	-1	0	1	1	0	2	

X₂ - Entering variable ; d₁⁻ - leaving variable

U-4
③

CB _i	C _j	B.V	X ₁	X ₂	d ₁ ⁻	d ₁ ⁺	d ₂ ⁻	d ₃ ⁻	Solution	Ratio
0	X ₂		0	1	1	-1	-1	0	2	
0	X ₁		1	0	0	0	1	0	8	
P ₂	d ₃ ⁻		0	0	-1	1	1	1	2	
	Z _j		0	0	-P ₂	P ₂	P ₂	P ₂	2P ₂	
	C _j -Z _j		0	0	P ₁ +P ₂	P ₃ -P ₂	(2P ₂ -P ₂)	0		
P ₃			0	0	0	1	0	0	0	
P ₂			0	0	0	-1	1	0	2	
P ₁			0	0	1	0	0	0	0	

∴ All C_j-Z_j ≥ 0 so optimality reached.

where x₁ = 8, x₂ = 2; d₃⁻ = 2

S.T.C x₁ + x₂ + d₁⁻ - d₁⁺ = 10

$$8 + 2 + 0 - 0 = 10 \Rightarrow 10 = 10$$

We want to satisfy P₂ goal, we want to sacrifice goal P₁, we always give priority to P₁ goal. so we stop the problem.

UNIT-V QUEUING THEORY

①

Introduction:-

* A common situation occurring or happening in every life is that of queuing or waiting in a line.

* Queuing (waiting line) are usually seen at bus stops, ticket booths, doctor's clinic, bank counters and traffic lights etc.

→ Queues are also found in workshops where the machines wait to be repaired, in warehouse where items are wait to be used, incoming calls wait to mature in the telephone exchange, trucks wait to be unloaded, airplanes wait either to take off or land and so on.

These problems arising due to:

- Too much demand on the facilities.
- Number of customers exceeds to the number of service facilities.
- Service facilities do not work efficiently.
- Server takes more time than prescribed time to serve a customer.

Queuing Theory can be applied to a variety of operational situations where it is not possible to predict accurately the arrival rate (or time) of customers and service rate (or time) of service facility or facilities.

In Practical, it can be used to determine the level of service (either the service rate or the number of service facilities) that balances the following two conflicting costs:

① cost of offering the service.

② cost incurred due to delay in offering service.

The first cost is associated with the service facilities and their operation, and the second represents the cost of customer's waiting time.

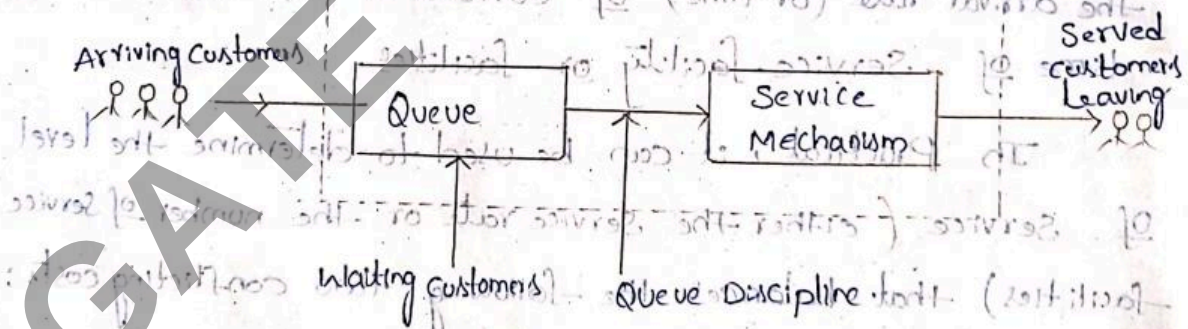
Many practical situations in which study of Queuing Theory can provide solution to waiting line problems are listed in the following table.

Station	Customers	Service Facility
1. Petrol Station	Automobiles	petrol pumps
2. Hospital	patients	Doctors/Nurses/Rooms.
3. Post office	Letters	sorting system
4. Airport	Aircrafts	Runways
5. Job Interview	Applicants	Interviewers

Queuing Structure / System:-

The mechanism of a queuing process is very simple. Customers arrive at a service counter and are attended to by one or more of the servers. As soon as a customer is served, it departs from the system. Thus a queuing system can be described as consisting of customers arriving for service, waiting for service if it is not immediate, and leaving the system after being served.

The General framework of a queuing system is shown below



Elements or Components of a Queuing System

The Basic elements of a queuing system are as follows

1. Input / Arrival process.
2. Queue Discipline.
3. Service Mechanism.
4. Capacity of the system.

1. Input process :- This element of queuing system is concerned

Input source can be described by following three factors:-
 a) Size of the source:- If a total number of potential customers requiring service are only few, then size of the input source is said to be finite. On the other hand, if potential customers requiring service are sufficiently large in number, then the input source is considered to be infinite. (3)

Also, the customers may arrive at the service facility in batches of fixed size or of variable size or one by one. In the case when more than one arrival is allowed to enter the system simultaneously (entering the system does not necessarily mean entering into service), the input is said to occur in bulk or in batches. Ships discharging cargo at a dock, families visiting restaurants, etc., are the examples of bulk arrivals.

b) Pattern of Arrivals:-

customers may arrive in the system at known (regular or otherwise) times or they may arrive in a random way. In case the arrival times are known with certainty, the queuing problems are categorized as deterministic models. On the other hand, if the time between successive arrivals (inter-arrival times) is uncertain, the arrival pattern is measured by either mean-arrival rate or inter-arrival time. These are characterised by the probability distribution associated with the random process. The most common stochastic queuing models assumed that arrival rate follow a poisson distribution and/or, the inter-arrival times follow an exponential distribution.

c) Customers Behaviour:-

It is also necessary to know the reaction of a customer upon entering the system. A customer may decide to wait no matter how long the queue becomes (patient customer), or if the queue is too long to suit him, may decide not to enter it (impatient customer). Machines arriving at the maintenance shop or a plant are examples of patient customers. For impatient customers,

i) If a customer decides not to enter the queue because of its length, he is said to have balked.

ii) If a customer enters the queue, but after some time loses patience and decides to leave, then he is said to have renege.

for position.

The final factor to be considered regarding the input process is the manner in which the arrival pattern changes with time. The input process which does not change with time is called a stationary input process. If it is time dependent then the process is termed as transient. (4)

2. Queue Discipline:-

The way of selecting customers for service according to certain rules known as the "queue discipline".

There are number of ways in which customers in the queue are served. Some of these are

(a) Static Queue Discipline.

(b) Dynamic Queue Discipline.

a) Static Queue Discipline:-

These are based on the individual customer's status in the queue. Few of such disciplines are:

i) First come first Served (FCFS):-

If the customers are served in the order of their arrival, then this is known as the first-come-first served queue discipline.

Ex:- Hospitals, Petrol stations.

ii) Last come first Served (LCFS):-

In this discipline the last customer arrived is served first because it reduces the time.

Example:- Cargo, movers and packers, production process.

b) Dynamic Queue Discipline:-

These are based on the individual customer attributes in the queue. Few of such disciplines are:

i) Service in Random order (SIRO):-

Under this rule customers are selected for service at random irrespective of their arrival in the service system.

ii) Priority Service (PS):-

Under this rule customers are grouped in priority classes on the basis of some attribute such as

Example:-

- i) The payment of telephone or electricity bills by cheque or cash.
- ii) Pre-emptive priority or emergency (PEP):

Under this rule, an important customer is allowed to enter into the system even if a customer with lower priority is already in service.

- iv) Non-pre-emptive priority (NPEP):

In this case an important customer goes ahead in the queue, but service is started immediately on the completion of the current service.

③ Service Mechanisms:-

The service mechanism is concerned with service time and service facilities. Service time is the time interval from the commencement of service to the completion of service. If there are infinite number of servers then all the customers are served instantaneously on arrival and there will be no queue.

If the number of servers is finite, then the customers are served according to a specific order. Further, the customers may be served in batches of fixed size or of variable size rather than individually by the same server, such as a computer with parallel processing or people boarding a bus. The service system in this case is termed as bulk service system.

In the case of parallel channels, "Fastest Server rule" (FSR) is adopted. For its discussion we suppose that the customers arrive before parallel service channels. If only one service channel is free, then incoming customer is assigned to free service channel. But it will be more efficient to assume that an incoming customer is to be assigned a server of largest service rate among the free ones.

Service facilities can be of the following types:

- a) Single queue - one server:- This is one queue-one service channel, where in the customer waits till the service point is ready to take him in for servicing.

Single queue Several Servers wherein the customers ready to take him in for servicing.

c) Several queues - one Server wherein there are several queues and the customer may join any one of these but there is only one service channel. (6)

d) Several Servers when there are several service channels available to provide service, much depends upon their arrangement. They may be arranged in parallel or in series or a more complex combination of both, depending on the design of the system's service mechanism.

By parallel channels, we mean a number of channels providing identical service facilities. Further customers may wait in a single queue until one of the service channels is ready to serve, as a barber shop where many chairs are considered as different service channels or customers may form separate queues in front of each service channel as in the case of super markets. For series channels, a customer must pass through all the service channels in sequence before service is completed. The situation may be seen in public offices where parts of the service are done at different service counters.

④ Capacity of the system:

The source from which customers are generated may be finite or infinite. A finite source limits the number of customers arriving for service, i.e., there is a finite limit to the maximum queue size. The queue can also be viewed as one with forced balking where a customer is forced to balk if he arrives at a time when queue size is at its limit. Alternatively, an infinite source is "forever" abundant as in the case of telephone calls arriving at a telephone exchange.

Notations in Queuing Theory

The notations used in the analysis of a queuing system are as follows:

① N - number of customers in the system (waiting and in

- 1) N = Maximum number of customers in a queueing system
- 2) S = Number of service channels (servers)
- 3) Q = Waiting line or queue
- 4) λ = Average (or expected) customer arrival rate per unit of time
- 5) μ = Average (or expected) customer service rate per unit of time
- 7) P = Traffic intensity (or) Service utilization factor (or) average (or expected) fraction of time for which server is busy
- $$P = \frac{\lambda}{\mu} = \frac{\text{Average service completion time } (1/\mu)}{\text{Average interarrival time } (1/\lambda)}$$
- 8) P_0 = Probability of no customer in the system = $(1-P)$
- 9) P_n = Probability of 'n' customer in the system
- 10) L_s = Average (or expected) number of customers in the system
- 11) L_q = Average number of customers in the queue
- 12) W_s = Average waiting time in the system
- 13) W_q = Average waiting time in the queue

Solution of Queuing Models

Two approaches are available for solving queuing models that is Mathematical and Simulation.

In Mathematical approach the actual arrival and service time distributions are approximated by one of the known mathematical distributions that is poisson, exponential etc., to describe expected value of various operational characteristics of the queuing process.

In this section, we will discuss various queuing models in which both arrivals and departures (those served) take place simultaneously with specific assumptions about the features of queuing systems such as: input source, arrival process, queue configuration, queue discipline and service process.

The simultaneous occurrence of arrivals and departures is also called birth and death process. This process helps in determining the probability distribution of the number of customers in the queuing system at a particular instance of time.

The probability distribution so obtained is then used to determine performance measures (operating characteristics) of various types of queuing models.

Types of models

1. Single - Server Queuing Model

(a) Model - I : $\{(M/M/1) : (\infty / FCFS)\}$ - unlimited Queue.

(b) Model - II : $\{(M/M/1) : (N / FCFS)\}$ - Limited Queue.

2. Multiple - Server Queuing Models

a. Model - III : $\{(M/M/s) : (\infty / FCFS)\}$ - unlimited Queue.

b. Model - IV : $\{(M/M/s) : (N / FCFS)\}$ - Limited Queue.

1. Single - Server Queuing Models

a) Model - I : $\{(M/M/1) : (\infty / FCFS)\}$ - unlimited Queue;

This model is based on certain assumptions about the queuing system:

1. Arrivals are described by Poisson probability distribution and come from an infinite population.
2. Single waiting line and each arrival waits to be served regardless of the length of the queue and no balking or reneging.
3. Queue discipline is "FCFS".
4. Single server and service time follow exponential distribution.
5. The average service rate is more than the average arrival rate.

If the probability of the system being empty (no customer) is denoted by P_0 and written as:

$$P_0 = 1 - \rho$$

then the probability of n customers in the system is denoted by P_n and written as:

$$P_n = \rho^n P_0$$

Performance Measures:-

1. $L_s = \frac{\lambda}{1 - \rho} = \frac{\lambda}{\mu - \lambda}$
2. $L_q = \rho \times L_s = \frac{\rho^2}{1 - \rho} = \frac{\lambda^2}{\mu(\mu - \lambda)}$

$$4. Wq = \frac{Lq}{\lambda} = Ws \times \rho = \frac{\lambda}{\mu(\mu-\lambda)} \quad \text{--- 100M}$$

5. probability that number of customer, 'n' in a system exceeds on given number 'k' as written as

$$P(n \geq k) = \rho^k \quad (9)$$

b) Model-II: $\{(M/M/1): (N/FCFS)\}$ = Limited Queue

In this model no more than N customers can be accommodated at any time in the system due to certain reasons. Thus any customer arriving when the system is already contains N customers does not enter the system and is lost.

Example: A finite queue may be arising due to physical constraint such as:

1. Emergency room in a hospital
2. one man barber shop with certain number of chairs for waiting customers etc.

Assumptions of this model are same as that of Model-I except that the length of the queue be limited.

If the number of customers $n=0,1,2,\dots,N$ ($P_0, P_1, P_2, \dots, P_N$).

then,

The probability of the system being empty (no customer) is denoted by P_0 and written as

$$P_0 = \frac{1-\rho}{1-\rho^{(N+1)}} \quad \left(\frac{\lambda}{\mu} \right) \left(\frac{\lambda}{\mu} \right) = \rho$$

The probability of 'n' customers in the system is denoted by P_n and written as:

$$P_n = \rho^n P_0 ; \rho \neq 1$$

Performance measures:

$$1. L_s = \frac{\rho}{1-\rho} - \frac{(N+1)\rho^{(N+1)}}{1-\rho^{(N+1)}} \quad \frac{1}{\mu} (1-\rho) = \frac{1}{\mu} (1-\rho)$$

$$2. L_q = L_s - \rho \left[\left(\frac{\lambda}{\mu} \right)^2 \left(\frac{\lambda}{\mu} \right) \frac{1}{\mu} \right] = \left(\frac{\lambda}{\mu} \right) \left(\frac{\lambda}{\mu} \right) \frac{1}{\mu}$$

$$3. W_s = \frac{L_s}{\lambda [1 - P_N]}$$

$$4. W_q = W_s - \frac{1}{\mu} = \frac{L_q}{\lambda [1 - P_N]} \quad \text{where } P_N = \rho^N P_0$$

27 Multiple - Server Queuing Models

Model - III : $\{ (M/M/S) : (\infty / F.I.S) \}$

In this case, instead of single server, there are multiple but identical servers in parallel to handle arriving customers.

For this queuing system, it is assumed that arrivals follow a poisson probability distribution: at an average rate of λ customers per unit of time and are served on a first-come first-served basis by any of the servers. The service times are distributed exponentially with an average of $1/\mu$ customers per unit of time. It is further assumed that only one queue is formed.

If the probability of the system being empty is denoted by P_0 and written as:

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n + \frac{1}{s!} \left(\frac{\lambda}{\mu} \right)^s \left(\frac{s\mu}{s\mu - \lambda} \right) \right]^{-1}$$

then, the probability of n customers in the system is denoted by P_n and written as:

$$P_n = \frac{\rho^n P_0}{n!}; \quad \rho = \frac{\lambda}{s\mu}$$

Performance Measures:

$$1. L_q = \left[\frac{(\lambda/\mu) \left(\frac{\lambda}{\mu} \right)^s}{(s-1)! (s\mu - \lambda)^2} \right] \frac{P_0}{1 - \rho} \quad L_s = \frac{\lambda}{\mu}$$

$$2. L_s = L_q + \frac{\lambda}{\mu}$$

$$3. W_q = \frac{L_q}{\lambda}$$

$$4. W_s = W_q + \frac{1}{\mu}$$

5. Probability that all servers are simultaneously busy

$$P(n \geq k) = \left[\frac{1}{s!} \left(\frac{\lambda}{\mu} \right)^s \left(\frac{s\mu}{s\mu - \lambda} \right) \right] P_0$$

Model - IV : $\{ (M/M/s) : (N/RCS) \}$ - Limited Queue

If the probability of the system being empty is denoted by P_0 and written as

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n + \sum_{n=s}^N \left(\frac{1}{s! s^{(n-s)}} \right) \left(\frac{\lambda}{\mu} \right)^n \right]^{-1}$$

Then,

The probability of n customers in the system is denoted by P_n and written as:

$$P_n = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n \right] P_0 ; \quad \rho = \frac{\lambda}{s\mu}$$

Performance Measures :-

$$1. L_q = \left[\frac{(s\rho)^s \rho}{s! (1-\rho)^2} \right] \left[1 - \rho^{(N-s+1)} - (1-\rho)(N-s+1) \rho^{(N-s)} \right] P_0$$

$$2. L_s = L_q + \frac{\lambda}{\mu} [1 - P_N]$$

$$3. W_q = \frac{L_q}{\lambda [1 - P_N]}$$

$$4. W_s = \frac{L_s}{\lambda [1 - P_N]}$$

where $P_N = P_n$

$$①. \rho = \frac{\lambda}{\mu}$$

$$\rho = \text{utilisation factor} = \frac{\text{Arrival Rate}}{\text{Service Rate}}$$

$$②. L_s (\text{Length of system}) = \frac{\rho}{1-\rho}$$

$$③. L_q (\text{Length of queue}) = L_s - \rho$$

$$④. W_s (\text{Average waiting time of a customer in system}) = \frac{L_s}{\lambda}$$

$$⑤. W_q (\text{Waiting time of a customer in queue}) = \frac{L_q}{\lambda}$$

$$⑥. P_0 (\text{zero number of customers in the system}) = 1 - \rho$$

$$⑦. P^n (\text{'n' number of customers in the system}) = \rho^n \cdot P_0$$

Problems

1) Arrivals at a Telephone Booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 3 minutes.

a) What is the Average length of the Queue that forms from time to time.

b) The Telephone department will install a second booth when convinced that an arrival would accept to have to wait at least 3 minutes for the phone by how much the flow of arrivals to be increased in order to justify second booth.

Sol Given that

$$\lambda = 1/10 ; \mu = 1/3$$

$$\rho = \frac{\lambda}{\mu} = \frac{1/10}{1/3} = 0.3$$

a) Average length of Queue

$$L_q = L_s - \rho$$

$$L_s = \frac{\rho}{1-\rho} = \frac{0.3}{1-0.3}$$

$$= \frac{0.3}{0.7} = 0.42$$

$$b) \lambda \mu = \frac{\lambda \mu}{\lambda}$$

$$= \frac{0.12}{1/10}$$

$$= 1.2$$

$$\mu = 1/3 ; \lambda = 0.12$$

$$\lambda \mu = \frac{\lambda \mu}{\lambda}$$

$$1.2 = \frac{0.12}{\lambda}$$

$$\lambda = 0.12$$

$$\lambda = \frac{0.12}{1.2}$$

$$\lambda = 0.1$$

first Telephone booth Arrival rate $\lambda = 1/10 = 0.1$

Difference b/w Second and first Telephone booth arrivals = 0.1017

$$\Rightarrow 0.1017 - 0.1 = 0.0017$$

2). In a Super market the average Arrival rate of a customer is 10 for Every 30 minutes following poisson process. The average time taken by cashier to a list and calculate the customers purchase is 2.5 minutes following exponential distribution. what is the probability of Queue length exceeds 6.
b) what is the expected time spent by a customer in the system

Sol

Given that

$$\text{Arrival rate } \lambda = 10/30 = 1/3 = 0.3333$$

$$\text{Service rate } \mu = 1/2.5 = 0.4$$

$$\rho = \frac{\lambda}{\mu} \Rightarrow \frac{0.3333}{0.4}$$

$$\rho = 0.8333$$

Length of Queue exceeds 6 is

$$\rho^6 = (0.8333)^6$$

$$\rho^6 = 0.3347$$

$$L_s = \frac{L_s}{\lambda} = \frac{0.8333}{1 - 0.8333} = 4.9988$$

$$L_s = \frac{4.9988}{0.8333} = 5.9979$$

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3) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes, calculate the following

- The mean queue size
- The probability that the queue size exceeds 10.

If the input of trains increase to an average 33 per day what will be the change in (a) & (b)

Sol. Given that

$$\lambda = \frac{30}{24 \times 60}$$

$$\lambda = \frac{30}{24 \times 60} = \frac{1}{48} = \frac{1}{12} \text{ hr}^{-1}$$

$$\mu = \frac{1}{36} \text{ hr}^{-1}$$

$$\rho = \frac{\lambda}{\mu} = \frac{1/48}{1/36} = 0.75$$

$$E(n) = \frac{\rho}{1 - \rho} = \frac{0.75}{1 - 0.75} = 3 \text{ trains}$$

$$P(Z \geq 10) = \rho^{10} = (0.75)^{10} = 0.06$$

When the input increase to 33 per day we'll have

$$\lambda = \frac{33}{24 \times 60} = \frac{33}{1440} = 0.023 \text{ hr}^{-1}$$

$$\mu = \frac{1}{36} \text{ hr}^{-1}$$

$$\rho = \frac{\lambda}{\mu} = \frac{0.023}{1/36} = 0.83$$

$$\text{Then we get } E(n) = \frac{\rho}{1 - \rho} = \frac{0.83}{1 - 0.83} = 4.9 \text{ (or) 5 trains}$$

$$P(Z \geq 10) = \rho^{10} = (0.83)^{10} = 0.1552 \Rightarrow \approx 0.2$$

4) In the production shop of a company the breakdown of the machines is 3 per hour. The cost of hiring an average rate of 3 machines per hour. Breakdown time at one machine cost Rs. 40 per hour to the company. There are two choices before the company for hiring the repairmen. One of the repairmen is slow but cheap, the other fast and expensive. The slow cheap repairmen demands Rs. 20 per hour and will repair the breakdown machines exponentially at the rate of 4 per hour. The fast expensive repairmen demands Rs. 30 per hour and will repair machines exponentially at an average rate of 6 per hour. Which repairmen should be hired.

Sol $\lambda = 3$ machines per hour
 Breakdown machine cost
 Rs. 40 per hour (1 machine)

cheap - slow: = Rs. 20 per hour, rate 4 per hour

fast - expensive = Rs. 30 per hour, rate 6 per hour

Case - I

$$\lambda = 3$$

$$\mu = 4$$

$$\rho = \frac{\lambda}{\mu} = \frac{3}{4} = 0.75$$

Average downtime of a machine

$$\frac{1}{\mu - \lambda} = \frac{1}{4 - 3} = \frac{1}{1} = 1 \text{ hour}$$

$\therefore$ The downtime of 3 machines that arrive in hour = $1 \times 3 = 3 \text{ hours}$

$$\text{Total cost} = (3 \times 40) + (3 \times 20) = 120 + 60 = \text{Rs. } 180/-$$

Case - II

$$\lambda = 3 ; \mu = 6$$

$$\rho = \frac{\lambda}{\mu} = \frac{3}{6} = 0.5$$

Average downtime of a machine

$$\frac{1}{\mu - \lambda} = \frac{1}{6 - 3} = \frac{1}{3} = 0.333 \text{ hours}$$

$\therefore$ The downtime of 3 machines that arrive in 0.333×3

$$\text{Total cost} = (1 \times 40) + (1 \times 30) = 40 + 30 = \text{Rs. } 70/-$$

Case study SIMULATION
 Simulation is a representation of reality through the use of model (or) other device which will react in the same manner as reality under a given set of conditions.

Mod.

Applications of Simulation:-

- 1) Monte-carlo Simulation -
- 2) Simulation on Inventory problems.
- 3) Simulation on queueing system -
- 4) Simulation on maintenance problems.
- 5) Simulation on investment and Budgeting.
- 6) Simulation on job sequencing.

Monte-carlo Simulation problem

- 1) The occurrence of rain in a city on a day is dependent upon weather (or) not it rained on the previous day. If it rained on the previous day the rain distribution is.

Event probability

Event	No rain	1cm rain	2cm	3cm	4cm	5cm
probability	0.50	0.25	0.15	0.05	0.03	0.02

- 2) If did not rain on the previous day the rain distribution is

Event	No rain	1cm	2cm	3cm
probability	0.75	0.15	0.06	0.04

Simulate the cities weather for 10 days and determine by simulation the total days without rain as well as the total rain-fall during the period. Use the following random numbers for the simulation.

67, 63, 39, 55, 29, 78, 70, 6, 78, 76.

Assume that for the first day of the simulation it had not rained the day before.

We simulate the cities weather with and without rainfall in the following steps

Step ① - Previous day rain Distribution

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Event	probability	Commulative Probability	Random Number Range
No rain	0.50	0.50	0 - 49
1cm	0.25	0.75	50 - 74
2cm	0.15	0.90	75 - 89
3cm	0.05	0.95	90 - 94
4cm	0.03	0.98	95 - 97
5cm	0.02	1.00	98 - 99

Step ② - previous Day No rain Distribution

Event	probability	Commulative probability	Random number range
No rain	0.75	0.75	0 - 74
1cm	0.15	0.90	75 - 89
2cm	0.06	0.96	90 - 95
3cm	0.04	1.00	96 - 99

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Step ③ - Simulate the 10 days for random numbers

Day	Random Number	Events
1	67	No rain
2	63	No rain
3	39	No rain
4	55	No rain
5	29	No rain
6	78	1cm
7	70	No rain
8	06	No rain
9	78	1cm
10	76	1cm

During the simulated period don't rain on previous days is 7 days.

Total rain days = 3 days = 3cm

The Monte-Carlo method is a simulation in which statistical distribution functions are created by using a series of random numbers.

Procedure:-

Step ① - clearly define the problem.

- a) Identify the objectives of the problem.
- b) Identify the main factors which have the greatest effect on the objectives of the problem.

Step ② - construct an appropriate model.

① Specify the variables and parameters of the model.

② Formulate the appropriate decision rules, that is state the conditions under which the experiment is to be performed.

③ Identify the type of distribution that will be used - Models use either theoretical distributions or empirical distributions to state the patterns of occurrence associated with the variables.

④ Specify the manner in which time will change.

⑤ Define the relationship between the variables and parameters.

Step ③ - Prepare the model for experimentation.

① Define the starting conditions for the simulation.

② Specify the number of runs of simulation to be made.

Step ④ - Using steps 1 to 3, experiment with the model.

① Define the coding system that will correlate the factors defined in step ① with the random numbers to be generated for the simulation.

② Select a random number generator and create the random numbers to be used in the simulation.

③ Associate the generated random numbers with the factors identified in step ① and coded in step ④.

Step ⑤ - Summarize and examine the results obtained in step ④.

Step ⑥ - Evaluate the results of the simulation.

Step ⑦ - Formulate proposals for advice to management on the course of action to be adopted and modify the model, if necessary.

Many of the Inventory problems, especially storage problems, cannot be solved analytically because of the complex nature of the distribution followed by demand or supply. It is however, possible to get the solution by using Simulation Techniques.

- * The Basic approach would be to determine the Probability distribution of the input and output functions from the past data and run the Inventory system artificially by generating the future observations on the assumption of the same distributions.
- * Subsequently, the decision making regarding the optimization problems would be made by the trial-and-error Method.
- * The Artificial Samples for future can be generated with the help of random numbers.
- * In Inventory control, the reorder point is to be chosen with consideration for the demand during lead time to provide adequate service to customers.
- * If both the lead time and demand of Inventory per unit of time, are random variables, then Simulation technique can be used to investigate the effect of different inventory policies.
- * We illustrate the use of simulation in the study of Inventory problems through some sample problems.

Bright Bakery keeps stock of a popular Brand of cake.
 Previous experience indicated the daily demand is given here

Daily Demand	0	10	20	30	40	50
probability	0.01	0.20	0.15	0.50	0.12	0.02

Consider the following sequence of random numbers
 48, 78, 19, 51, 56, 77, 15, 14, 68, 9
 using this sequence simulate the demand for the next 10 days.
 find out the stock situation if the owner of the Bakery decides
 to make 30 cakes everyday. Also estimate the daily average
 demand for the cakes on the basis of simulated data.

sol First simulate the demand of Bakes with the following
 distribution.

Daily Demand	probability	Cumulative Probability	Random Number Range
0	0.01	0.01	0 - 0
10	0.20	0.21	1 - 20
20	0.15	0.36	21 - 35
30	0.50	0.86	36 - 85
40	0.12	0.98	86 - 97
50	0.02	1.00	98 - 99

simulate the demand for 10 days with Random Numbers

Days	Random Number	Demand	stock
1	48	30	0
2	78	30	0
3	19	10	20
4	51	30	0
5	56	30	0
6	77	30	0
7	15	10	20
8	14	10	20
9	68	30	0
10	9	10	20

$$\text{Average Demand} = \frac{\text{Total Demand}}{\text{No. of Days}}$$

$$= \frac{270}{10}$$

$$= 27 \text{ Per day}$$

Remaining stock = 80

2) The Automobile company manufactures around 150 scooters. The daily production varies from 146 to 154 depending upon the availability of raw material and other working conditions.

Production Per day	146	147	148	149	150	151	152	153	154
Probability	0.04	0.09	0.12	0.14	0.11	0.10	0.20	0.12	0.08

The finished scooters are transported in a specially arranged lorry Accommodating 150 scooters using following random Numbers.

80, 81, 76, 75, 64, 43, 18, 26, 10, 12, 65, 68, 69, 61, 57

Simulate the process to find out:

a) What will be the Average number of scooter waiting in the factory.

b) What will be the Average number of Empty Space in the lorry.

Sol The random numbers are established as in Table below

Random Number coding

Production	probability	cummulative probability	Random Number Range
146	0.04	0.04	01-3
147	0.09	0.13	4-12
148	0.12	0.25	13-24
149	0.14	0.39	25-38
150	0.11	0.50	39-49
151	0.10	0.60	50-59
152	0.20	0.80	60-79
153	0.12	0.92	80-91
154	0.08	1.00	92-99

Based on the 15 random numbers given, we simulate the production per day in the table below

80	81	76	75	64	43	18	26	10	12	65	68	69	61	57
80	81	76	75	64	43	18	26	10	12	65	68	69	61	57
80	81	76	75	64	43	18	26	10	12	65	68	69	61	57

S.NO	Production Random Number	Production Per day	No. of Scooters waiting in the	Empty Space in Lorry
1	80	153	3	0
2	81	153	3	0
3	76	152	2	0
4	75	152	2	0
5	64	152	2	0
6	43	150	0	0
7	18	148	-	2
8	26	149	-	1
9	10	147	-	3
10	12	147	-	3
11	65	152	2	0
12	68	152	2	0
13	69	152	2	0
14	61	152	2	0
15	57	151	1	0

a) Average number of Scooters waiting = $\frac{\text{No. of Scooters waiting}}{\text{Total no. of days}}$

$$= \frac{21}{15} = 1.4$$

b) Average no. of Empty space in the lorry = $\frac{\text{No. of Empty Space}}{\text{Total no. of days}}$

$$= \frac{9}{15}$$

Saturday
31/3/2018

3. The Management of ABC companies is considering the question of marketing a new product. The fixed cost required in the project is 4,000/-. Three factors are uncertain i.e., Selling price, variable cost and the sales volume. The product has a life of only one year. The management has the data on these

Selling Price (RS)	Probability	Variable Cost (RS)	Probability	Sales Volume	Probability
3	0.2	1	0.3	2000	0.3
4	0.5	2	0.6	3000	0.3
5	0.3	3	0.1	5000	0.4

Consider the following sequence of 30 random numbers

81, 33, 60, 04, 46, 31, 67, 25, 24, 10, 40, 02, 39, 68, 08, 59, 66, 90, 19, 64, 79, 31, 86, 68, 82, 89, 25, 11, 98, 16

using the sequence first three random numbers for the first trial. Simulate the average profit of the above project on the basis of 10 trials.

Sol first simulate the selling price, variable cost, sales volume for selling price

Selling price	probability	Cumulative Probability	Random number Range
3	0.2	0.2	0 - 19
4	0.5	0.7	20 - 69
5	0.3	1.0	70 - 99

for variable cost

Variable cost	probability	Cumulative Probability	Random number Range
1	0.3	0.3	0 - 29
2	0.6	0.9	30 - 89
3	0.1	1.0	90 - 99

for sales volume

Sales Volume	probability	Cumulative Probability	Random Number Range
2000	0.3	0.3	0 - 29
3000	0.3	0.6	30 - 59
5000	0.4	1.0	60 - 99

Step 1 - Simulate the average profit for the project with 10 random numbers. The project has a life of one year. The management is considering the project on the basis of the average profit.

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Trials	Random Selling price Number	Selling Price	Random Variable cost	Variable cost	Random number for sales volume	Sales volume	Profit
1	81	5	04	1	67	5000	16000
2	32	4	46	2	25	2000	0
3	60	4	31	2	24	2000	0
4	10	3	39	2	59	3000	-1000
5	40	4	68	2	66	5000	6000
6	2	3	08	1	90	5000	6000
7	12	3	31	2	82	5000	1000
8	64	4	86	2	89	5000	6000
9	79	5	68	2	25	2000	2000
10	11	3	98	3	16	2000	-4000
						32000	32000

$$\text{profit} = \left[(\text{Selling price} - \text{variable cost}) \times \text{sales volume} \right] - \text{fixed cost}$$

$$\text{Avg profit} = \frac{\text{Total Profit}}{\text{No. of Trials}} = \frac{32000}{10} = 3200$$

4) The management of ABC companies is considering the question of marketing a new product. The fixed cost required in the project is 4000. Three factors are uncertain i.e., selling price, variable cost and the sales volume. The product has a life of only one year. The management has the data on these

Selling price	Probability	variable cost (RS)	probability	Sales volume	Probability
3000	0.2	1	0.3	2000	0.3
4000	0.5	2	0.6	3000	0.3
5000	0.3	3	0.1	5000	0.4

Consider the following sequence of 30 random numbers

81, 32, 60, 04, 46, 31, 67, 25, 24, 10, 40, 02, 39, 68, 08, 59, 66, 90

12, 64, 79, 31, 86, 68, 82, 89, 25, 11, 98, 16

Using the sequence first random number for the first Trial, Second random number for the second trial and (third) random number for the Third Trial. Simulate the average profit of the above project on the basis of 10 trials.

1) first simulate the selling price, variable cost and sales

Selling price	probability	Cummulative probability	Random number Range
3	0.2	0.2	0 - 19
4	0.5	0.7	20 - 69
5	0.3	1.0	70 - 99

Variable cost			
Variable cost	probability	Cummulative probability	Random number Range
1	0.3	0.3	0 - 29
2	0.6	0.9	30 - 89
3	0.1	1.0	90 - 99

Sales volume			
Sales volume	probability	Cummulative probability	Random number Range
2000	0.3	0.3	0 - 29
3000	0.3	0.6	30 - 59
5000	0.4	1.0	60 - 99

Step 3 - Simulate the average profit for the project with random numbers.

Trails	Random Number	Selling Price	Random Number variable cost	Variable cost	R.N. for sales volume	Sales Volume	Profit
1	81	5	32	2	60	5000	11000
2	4	3	46	2	31	3000	-1000
3	67	4	25	2	24	2000	2000
4	10	3	40	2	2	2000	-2000
5	39	4	68	2	08	2000	0
6	59	4	66	2	90	5000	6000
7	12	3	64	2	79	5000	1000
8	31	4	86	2	68	5000	6000
9	82	5	89	2	25	2000	2000
10	11	3	98	3	16	2000	-4000

$$\text{Profit} = (\text{Selling Price} - \text{variable cost}) \times \text{sales volume} - \text{fixed cost}$$

$$\text{Avg profit} = \frac{21000}{10} = 2100$$

measures of effectiveness, such as Queue length, Average waiting time etc. When the distribution of Inter Arrival time and Service times are known. If costs be assigned to waiting time of customers and Idle time of the service facility, the problem of establishing a proper balance b/w these costs can be determined.

However many queuing problems cannot be solved explicitly by Analytical methods. In such cases the only possible method of solution is to simulate the experiment. We illustrate the use of simulation in the study of queues through some sample problems.

- 1) Dr. Strong is a Dentist who schedules all her patients for 30 mins appointments. Some of the patients take more or less than 30 mins depending on the type of Dental work to be done. The following summary shows the various categories of work, their probabilities and the time actually to complete the work

Category	Time Required (mins)	Probability of Category
Filling	45 mins	0.40
Crown	60 mins	0.15
Cleaning	15 mins	0.15
Extraction	45 mins	0.10
Checkup	15 mins	0.20

Simulate the dentist's cleaning for 4 hrs and Determine the Average waiting time for the patients as well as Idleness of doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival time starting at 8:00 AM.

use the following random numbers handling the above problem

40, 82, 11, 34, 25, 66, 17, 79

Sol Allotment of random numbers for category of work for the given probabilities is shown in below.

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	Probability	Cumulative	Random number range
filling	0.40	0.40	0 - 39
crown	0.15	0.55	40 - 54
cleaning	0.15	0.70	55 - 69
Extraction	0.10	0.80	70 - 79
checkup	0.20	1.0	80 - 99

future event

Patient NO	Schedule Arrival	Random number	category	Service time
1	8:00	40	crown	60 mins
2	8:30	82	checkup	15 mins
3	9:00	11	filling	45 mins
4	9:30	34	filling	45 mins
5	10:00	25	filling	45 mins
6	10:30	66	cleaning	15 mins
7	11:00	17	filling	45 mins
8	11:30	79	Extraction	45 mins

Simulation of Queuing

Patient	Arrival	Service Start	Service Duration	Service ends	Waiting in minutes	Idle time of doctor
1	8:00	8:00	60	9:00	-	-
2	8:30	9:00	15	9:15	-	-
3	9:00	9:15	45	10:00	30 mins	-
4	9:30	10:00	45	10:45	15 mins	-
5	10:00	10:45	45	11:30	30 mins	-
6	10:30	11:30	15	11:45	45 mins	-
7	11:00	11:45	45	12:30	60 mins	-
8	11:30	12:30	45	1:15	45 mins	-
					285	

Average waiting

Time = 285

8 = 35.625

36 mins

Idle time of doctor zero mins.

of interarrival duration and service duration in a single channel queuing system. using the random number table below, stimulate the queue system behaviour for a period of 60 mins and estimate the probability of the service being ideal and then mean time spent by a customer waiting for service.

Inter arrival time		Service time	
Minutes	Probability	Minutes	probability
2	0.15	1	0.10
4	0.23	3	0.22
6	0.35	5	0.35
8	0.17	7	0.23
10	0.10	9	0.10

Random numbers start at North West corner and proceed along the row. 9371, 1463, 7214, 1053, 2164, 8142, 8709, 9054, 13866, 1053, 2924, 1725, 1185, 6885, 9980, 5119, 4086, 3083, 5217, 7105. The queue process begin at 9:00 AM and estimate queue system for 9 customers.

Sol The cumulative probability distribution and random number interval for inter arrival and service times.

For Interarrival time

Minutes	Probability	Cumulative Probability	Random number Range
2	0.15	0.15	0 - 14
4	0.23	0.38	15 - 37
6	0.35	0.73	38 - 72
8	0.17	0.90	73 - 89
10	0.10	1.00	90 - 99

for Service time

Minutes	Probability	Cumulative Probability	R.N Range
1	0.10	0.10	10 - 9
3	0.22	0.32	10 - 31
5	0.35	0.67	32 - 66
7	0.23	0.90	67 - 89
9	0.10	1.00	90 - 99

A sample of 100 arrivals of a customer at a petrol station department is according to the following distribution

Time b/w arrivals (mins)	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Frequency	2	6	10	25	20	14	10	7	4	2

A study of time required to service customers, by adding at the bill, receiving, payments and placing packages yields the following distribution:

Time b/w service	0.5	1	1.5	2	2.5	3
Frequency	12	21	36	19	7	5

So estimate the average percentages of customer waiting time and average percentage of ideal time of the server by simulation for the next 10 arrivals. Use the random numbers

93, 22, 53, 64, 39, 7, 10, 63, 76, 35 and random numbers for the service time is 78, 76, 58, 54, 74, 92, 38, 70, 96, 92

sol

Arrival time

Arrival	Frequency	Probability	Cumulative Probability	Random number Range
0.5	2/100	0.020	0.02	0 - 1
1	6	0.060	0.08	2 - 7
1.5	10	0.10	0.18	8 - 17
2	25	0.25	0.43	18 - 42
2.5	20	0.20	0.63	43 - 62
3	14	0.14	0.77	63 - 76
3.5	10	0.10	0.87	77 - 86
4	7	0.07	0.94	87 - 93
4.5	4	0.04	0.98	94 - 97
5	2	0.02	1.00	98 - 99

Service	Frequency	Probability	Cumulative Probability	Random number Range
0.5	12/100	0.12	0.12	0 - 11
1	21	0.21	0.33	12 - 32
1.5	36	0.36	0.69	33 - 68
2	19	0.19	0.88	69 - 87
2.5	7	0.07	0.95	88 - 94
3	5	0.05	1.00	95 - 99

Arrival Time	probability	Cumulative probability	R.N Range
5	0.10	0.10	0 - 9
6	0.15	0.25	10 - 24
7	0.25	0.50	25 - 49
8	0.25	0.75	50 - 74
9	0.15	0.90	75 - 89
10	0.10	1.00	90 - 99

Service Time

Service Time	probability	Cumulative Probability	Random Number Range
4	0.15	0.15	0 - 14
5	0.20	0.35	15 - 34
6	0.30	0.65	35 - 64
7	0.25	0.90	65 - 89
8	0.10	1.00	90 - 99

R.N	Arrival Time	Inter Arrival Time	R.N	Service Time	Services		Waiting Time	
					start	end	customer	Server
85	9	9	69	7	9	16		A
88	5	14	70	7	14	21		
90	9	23	21	4	23	27		
38	7	33	45	6	33	39		
40	7	40	86	7	40	47		
35	7	47	93	8	47	55		
55	8	54	24	5	54	59		
43	7	62	73	7	62	69		
24	6	69	26	5	69	74		
8	5	75	96	8	75	83		
14	6	80	118	14	80	84		
12	6	86	86	7	86	93		
62	8	92	26	5	92	97		
99	10	100	33	5	100	105		
		110	97	8	110	118		

PP, SD, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100

FP, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100